EPFL Cours d'Electrotechnique I

2. Conventions et les symboles :

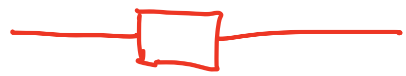
→ concepts → modèle

Ex : courant : i, I, $\hat{I}$, $\hat{i}$, $\bar{I}$, $\underline{I}$

unité : [A]

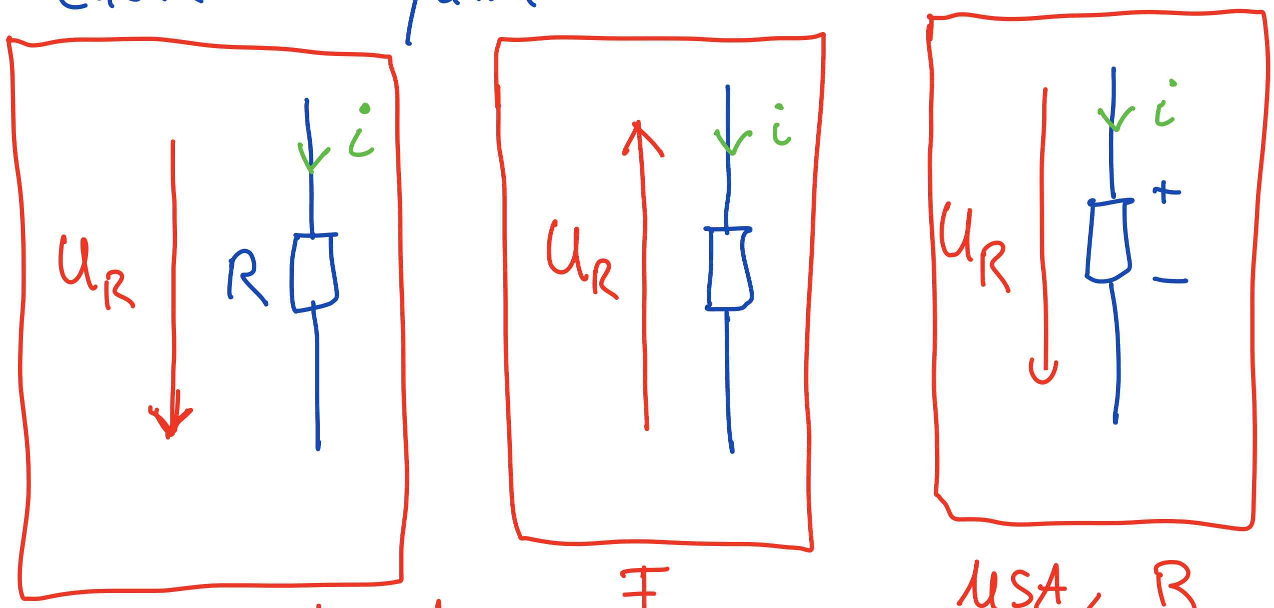
Relations : $U = R \cdot I$

$u = R \cdot i$

Dessin : 

Résistance

Choix à faire :

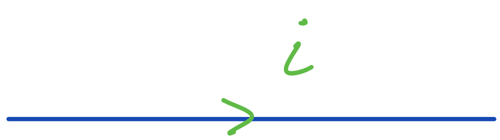


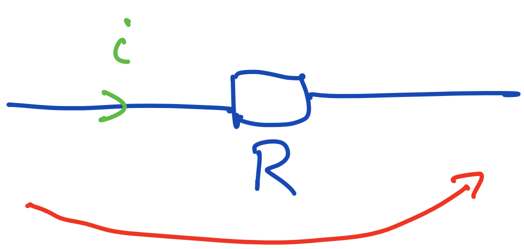
International — Fr — USA, B

convention moteur : choix

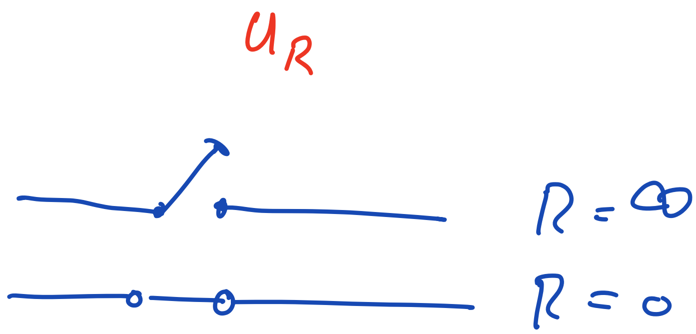
2.2 Représentation graphique :

Conducteur parfait :

Conducteur avec un courant : 

Élément : 

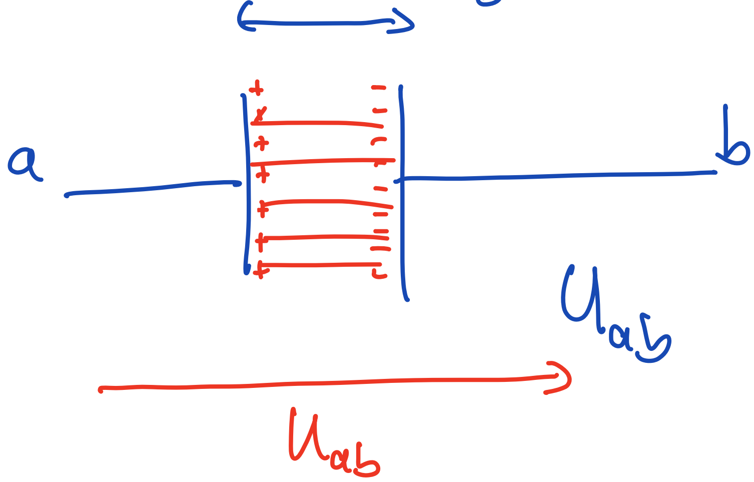
Interrupteur :



3. Lois fondamentales :

Différence de potentiel : Tension [Volt]

$$V_a - V_b = \int_b^a E\,dl = U_{ab} \quad [V]$$



3.2.19 La Capacité :

Définition : Charge électrique : Q

Capacité : $C = \dfrac{Q}{U_{ab}}$

Symbole : 

3.3 Courant électrique :

$$I = \frac{dQ}{dt} \quad [A]$$

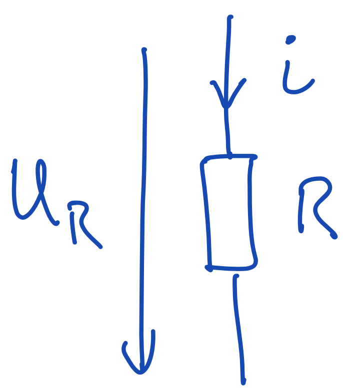
Densité de courant : $J \Rightarrow [A/m^2]$

3.3.4 Pertes Joule :

$$P = R \cdot I^2 \quad [W]$$

Récap :

Convention moteur :



→ Puissance positive

→ conv. moteur consommateur.

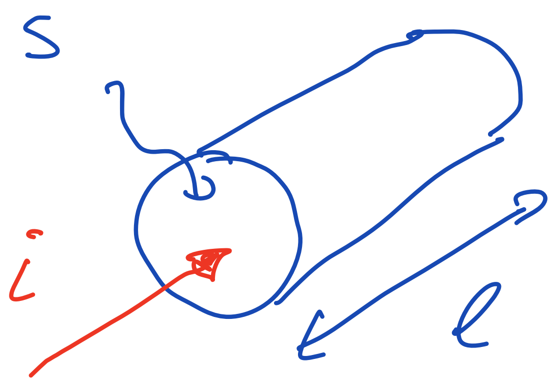
3.3.6 Définition de la résistance :

$$R_{ab} = \int_a^b \rho \cdot \frac{dl}{S}$$

dl ← longueur

S ← surface

ρ ← résistivité électrique [Ωm]



Si S est constante sur la longueur

$$R_{ab} = \frac{\rho \cdot l}{S} \quad [\Omega]$$

3.3.8 Loi d'Ohm :

$$U_{ab} = R_{ab} \, I$$

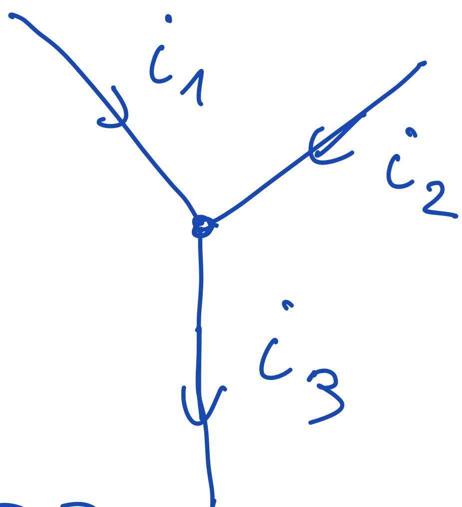
(courant et tension continue)

$$u_{ab} = R_{ab} \cdot i$$ (courant et tension variable)

3.3.11 Lois de Kirchhoff :

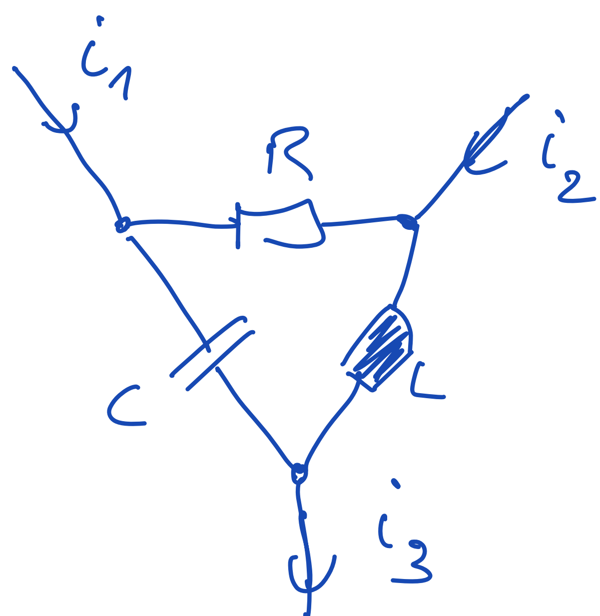
Noeud : Point de convergence d'au moins trois conducteurs

$$\boxed{\sum i_j = 0}$$



$$i_1 + i_2 - i_3 = 0$$

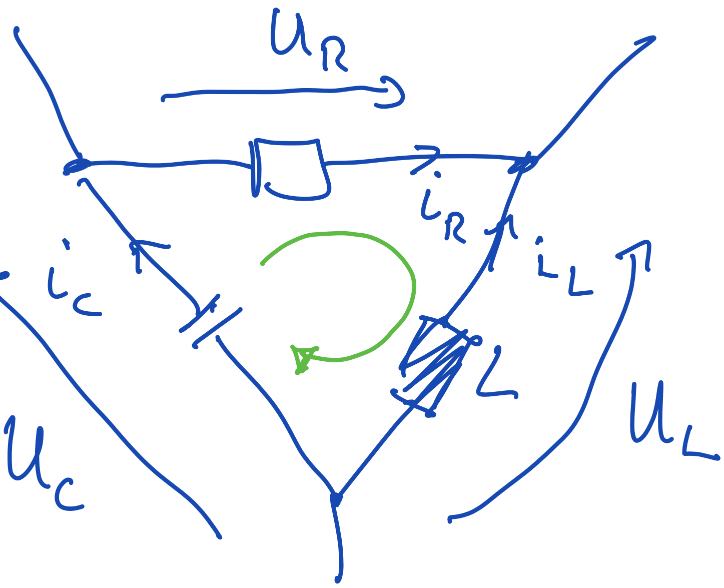
Noeud généralisé :



$$i_1 + i_2 - i_3 = 0$$

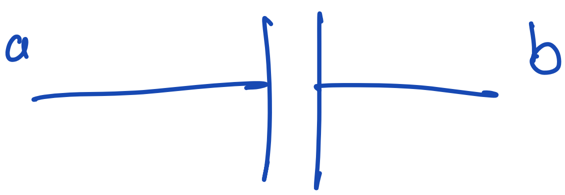
Maille : ensemble de branche partant d'un noeud pour y retourner

$\sum U_j = 0$



$$U_R - U_L + U_C = 0$$

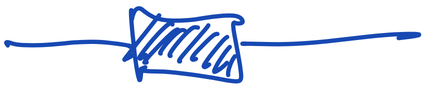
3.5 La Capacité :



$$C = \frac{Q}{U_{ab}} \qquad Q = \int i \, dt$$

$$u = \frac{1}{C}\int i\,dt$$

3.4 l'inductance :



L

L

$$u = L\,\frac{di}{dt}$$

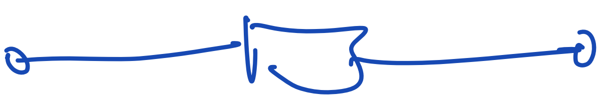
$$\overrightarrow{\text{Rot}}\,\vec{H} = \vec{J}$$

$$\overrightarrow{\text{Rot}}\,\vec{E} = -\frac{d\vec{B}}{dt}$$

L [H] Henry

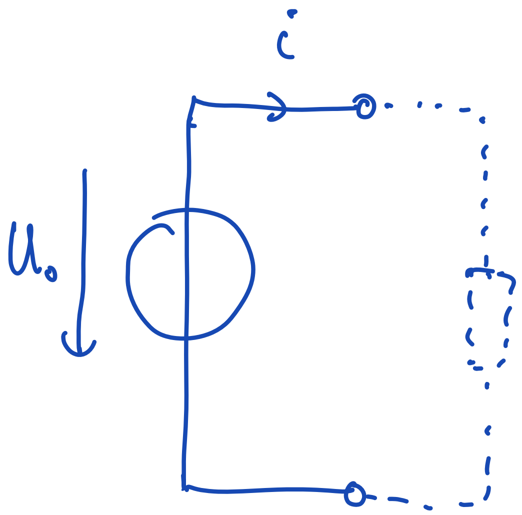
4. Eléments de circuit :

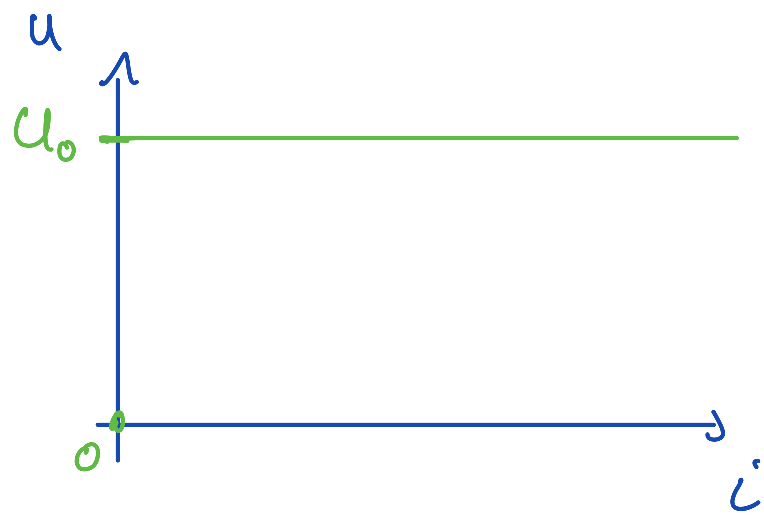
4.1 Dipôle : circuit qui possède 2 bornes



4.2 Sources de tension et de courant

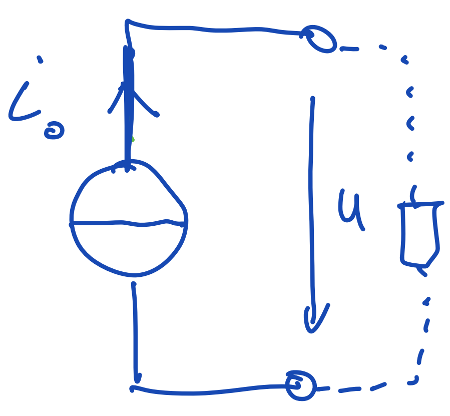
a) Source de tension idéale :

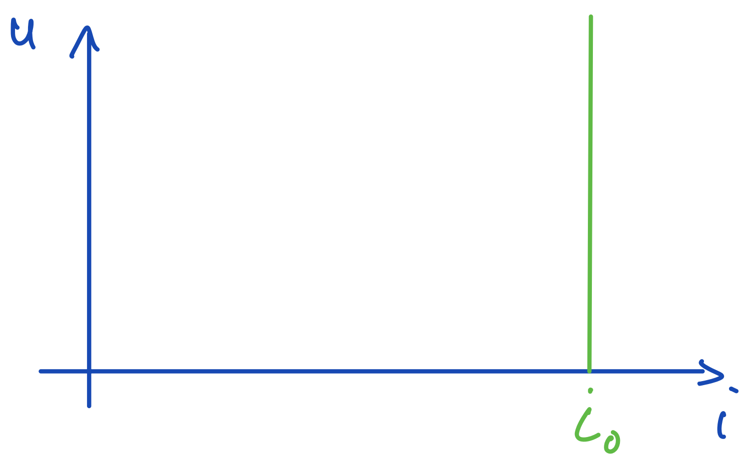




c'est un élément virtuel, idéal et inexistant dans la nature

b) Source de courant idéale :

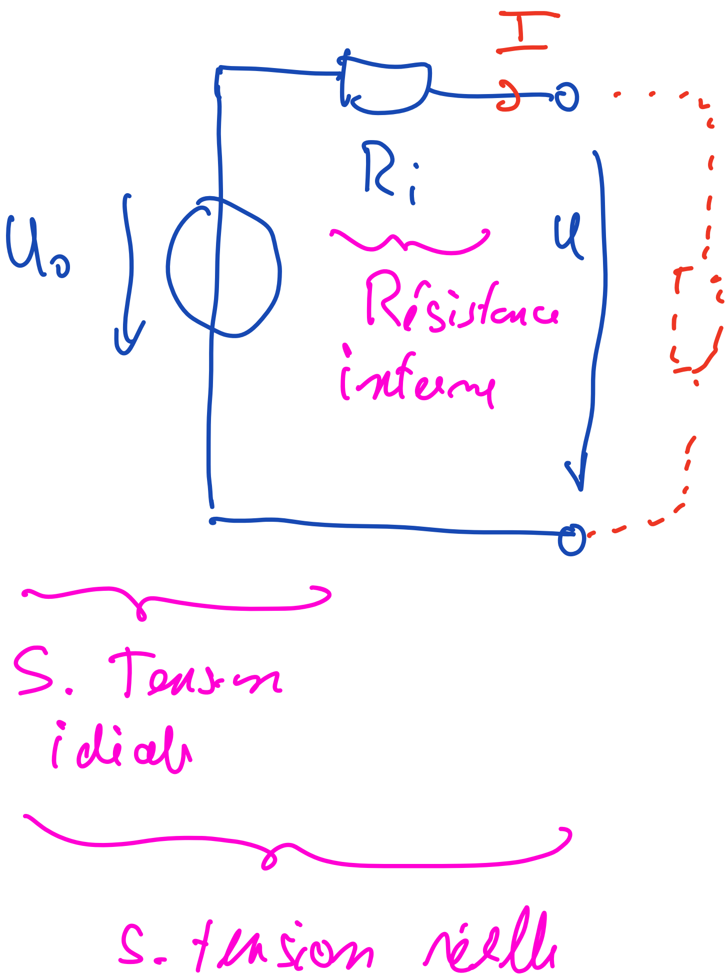




élément virtuel, inexistant dans la nature.

4.2.5 Source de tension réelle :

Déf :

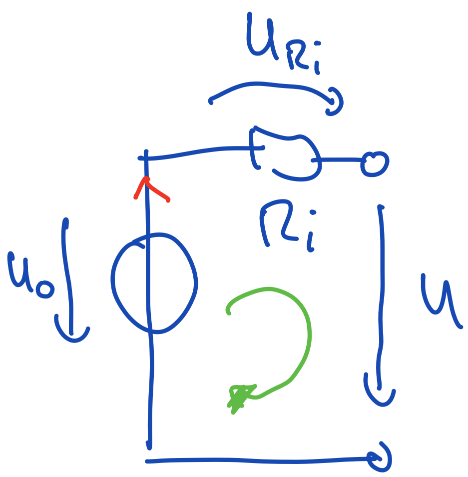


U_0 : Tension de la source idéale

Tension à vide

R_i : Résistance interne

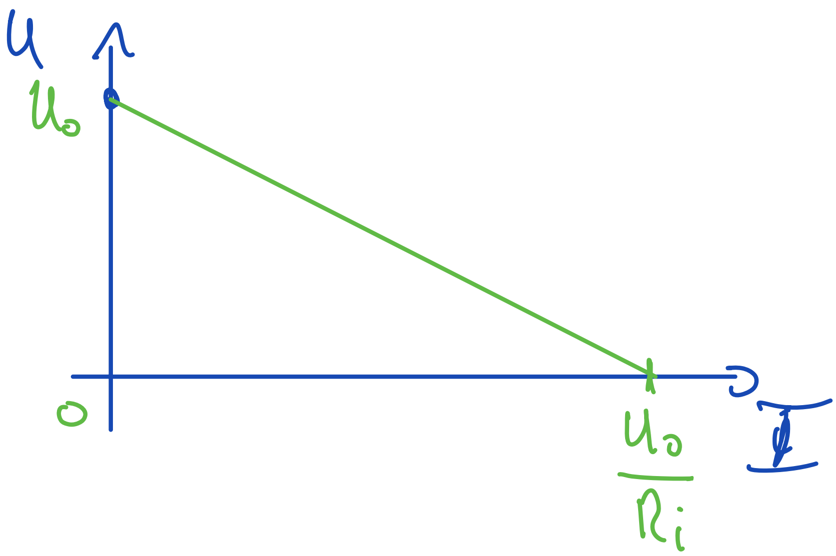
U : Tension de la source



$$\sum U = 0$$

$$-U_0 + \underbrace{U_{R_i}}_{=R_i \cdot I} + U = 0$$

$$U = U_0 - R_i \cdot I$$



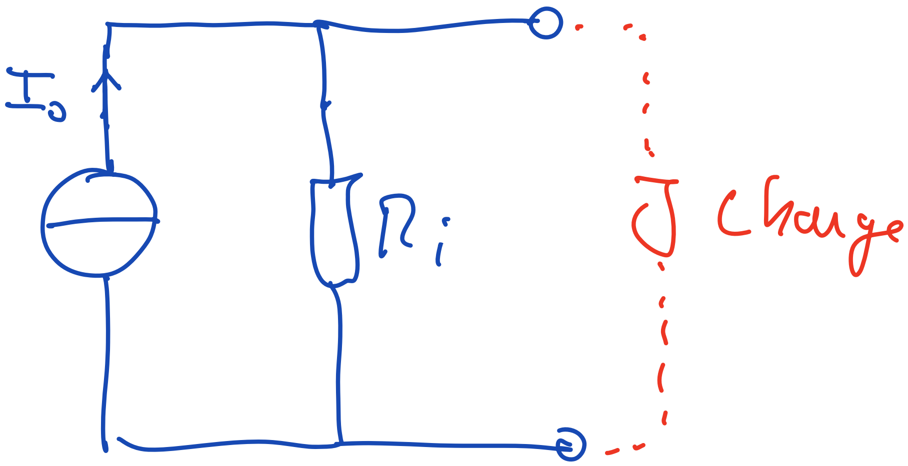
Courant max :

$U = 0$

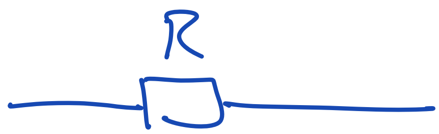
$0 = U_0 - R_i \cdot I_{cc}$

$$I_{cc} = \frac{U_0}{R_i}$$

4.2.6 Source de courant réelle :



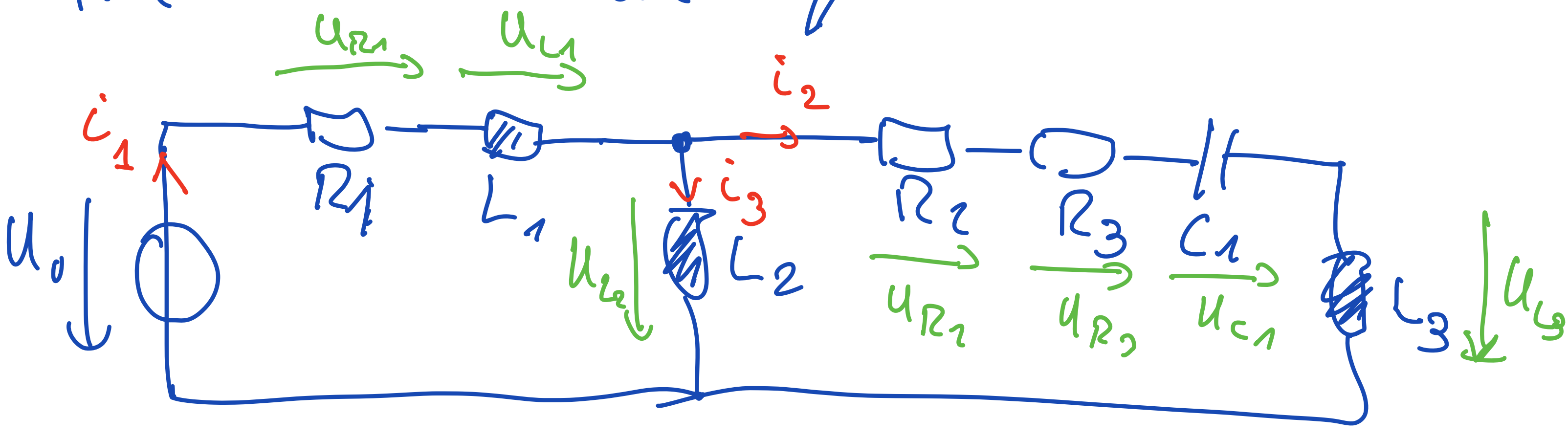
4.3 Élément de base :

Résistance 

Inductance 

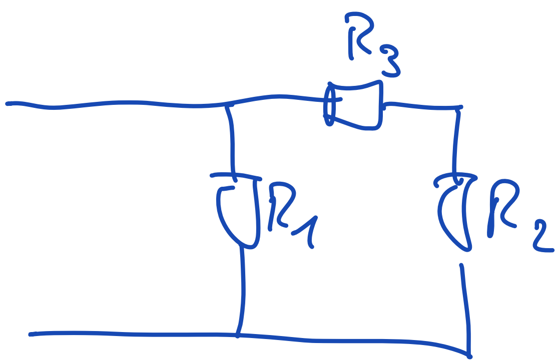
Capacité 

4.4 Schéma électrique :



Recap : Quiz :

4 : // → même tension aux bornes



R_1 n'est pas en // avec R_2 !

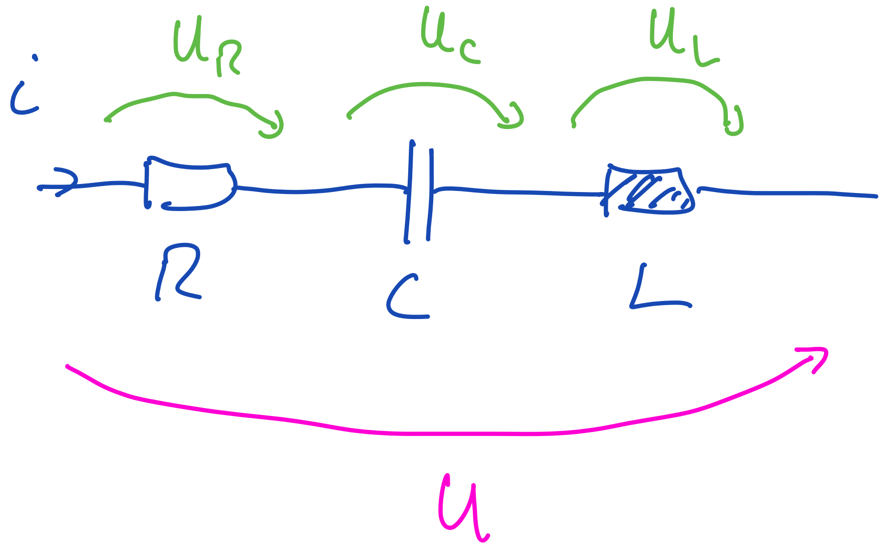
7 : Source idéale seule impossible.

8 : Source idéale est toujours constante

9 : Impossible de mettre des sources de courant en série.

5. Combinaison simple d'éléments linéaires

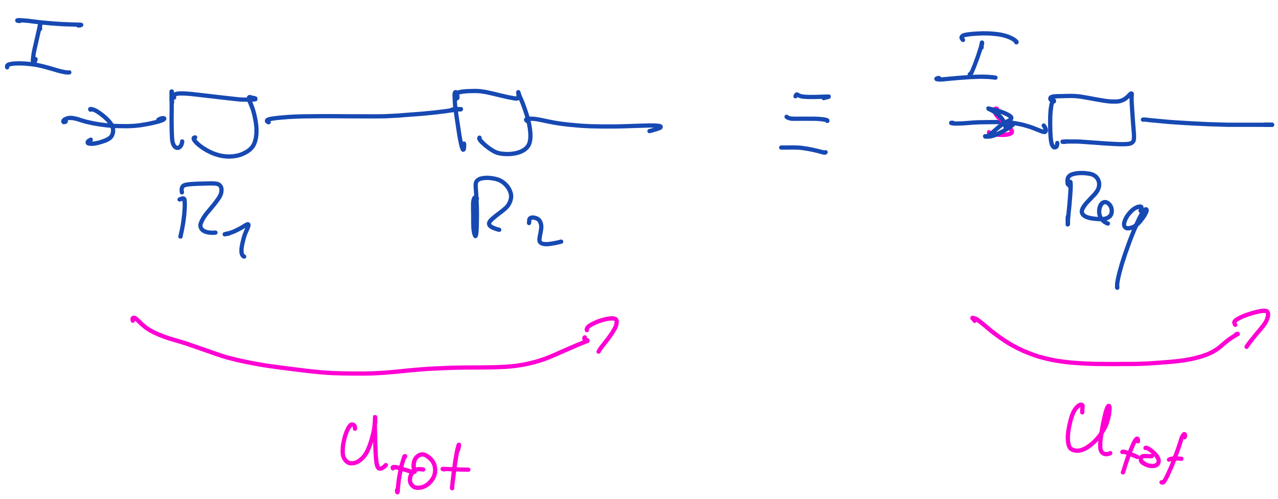
5.2 Mise en série :



Série : parcouru par le même courant

$i_R = i_L = i_C \rightarrow$ Série

5.2.2 Mise en série de la résistance



$$U_{tot} = U_{R_1} + U_{R_2} \qquad U_{tot} = R_{eq} \cdot I$$

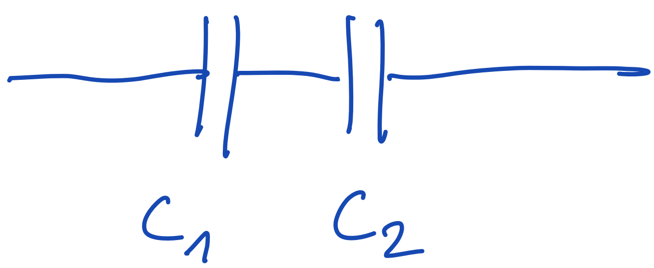
$$= R_1 I + R_2 I$$

$$= (R_1 + R_2) I = R_{eq} \cdot I$$

$$\Longrightarrow R_{eq} = R_1 + R_2$$

En série $R_{eq} = \sum_{K=1}^{n} R_K$ (n = nb de R)

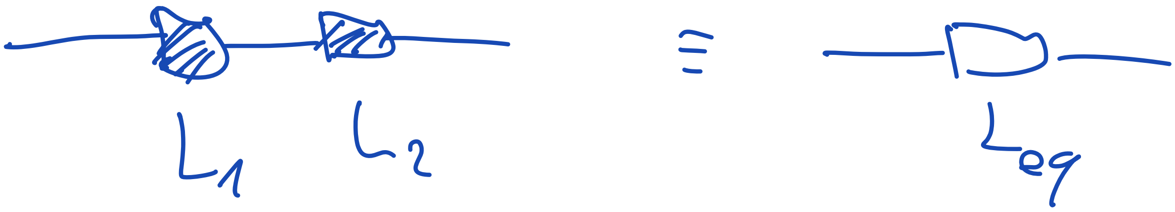
5.2.3 Mise en série des C



$\equiv$ $C_{eq} = ?$

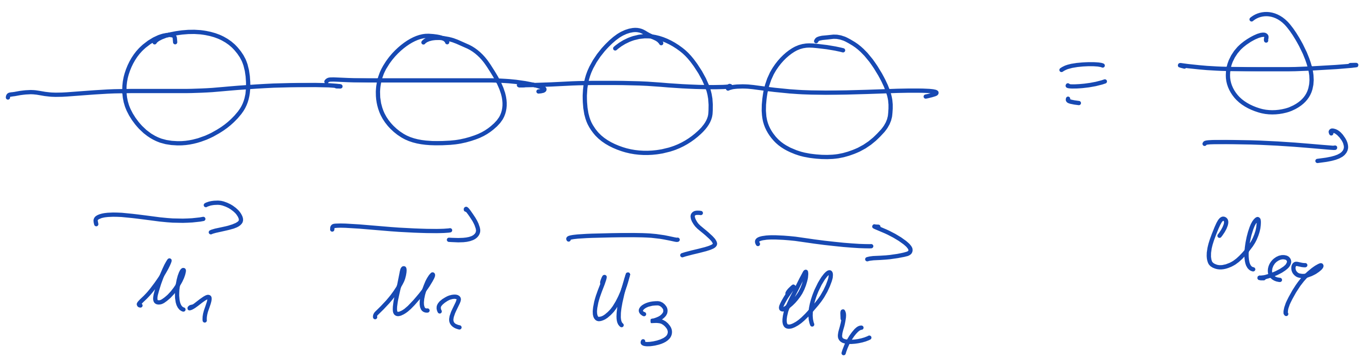
Série $C_{eq} = \dfrac{1}{\sum_{K=1}^{n} \dfrac{1}{C_K}}$ n = nb de C

5.2.6 Mise en série des L

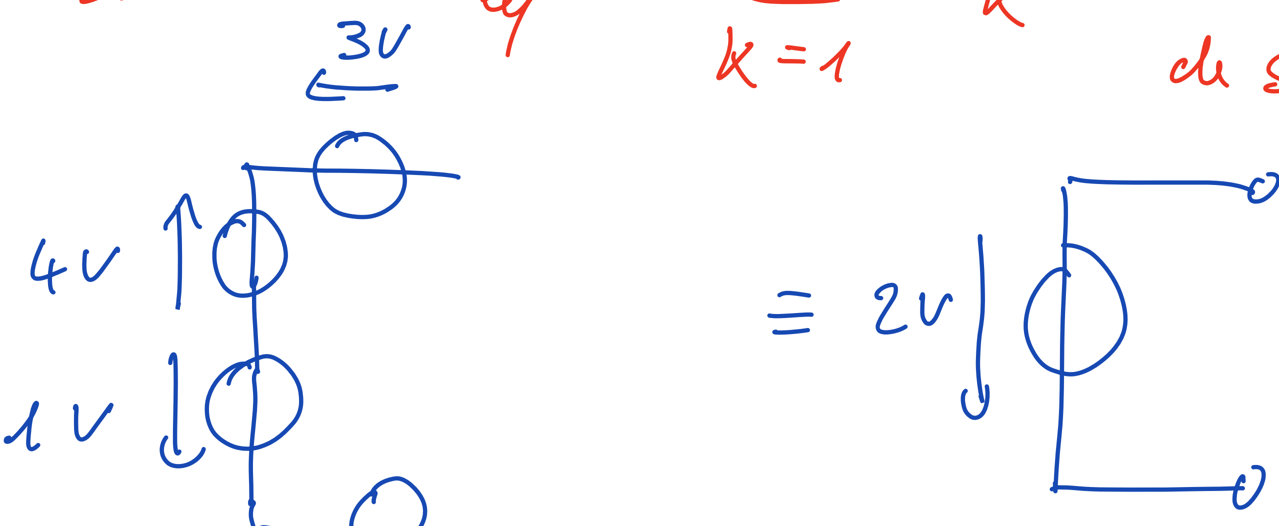


Série $L_{eq} = \sum_{k=1}^{n} L_k$ n = nb de L

5.2.7 Mise en série de source de tension

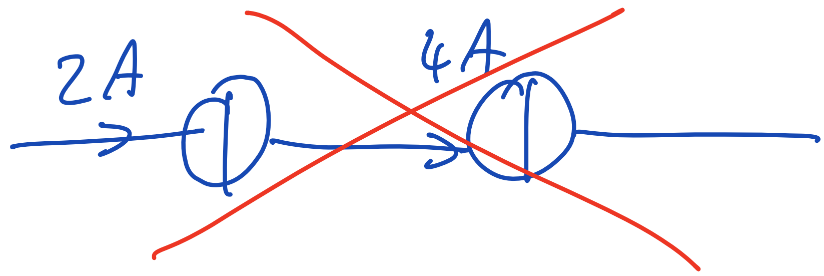


Série $U_{eq} = \sum_{k=1}^{n} U_k$ n = nb de source



$\overrightarrow{2U}$

5.2.9 Mise en série de source de courant

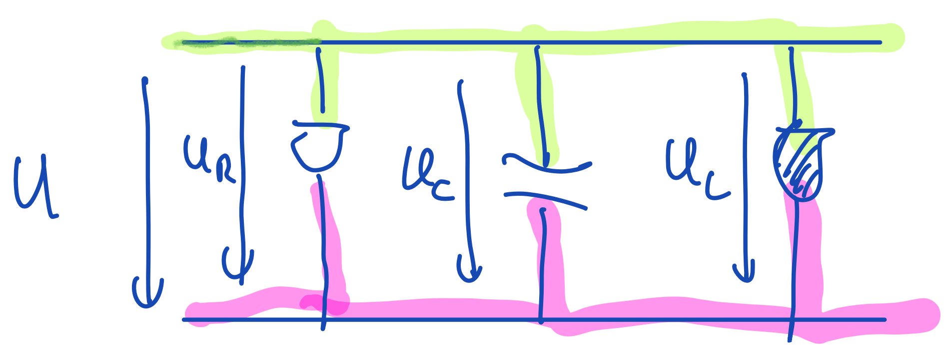


impossible

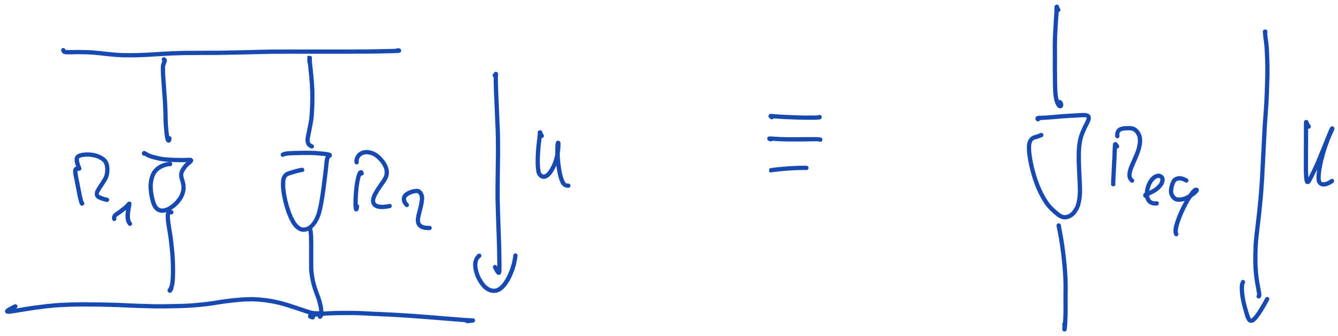
$\Rightarrow$ Impossible sauf si toutes les sources ont le même courant

5.3.2 Mise en // des R :

Définition : Toutes les bornes des éléments sont au même potentiel



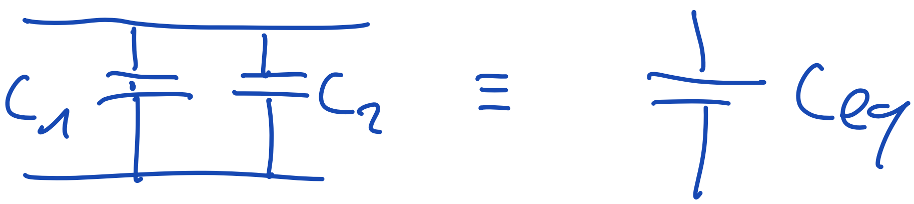
$$u_R = u_C = u_L$$



$$R_{eq} = \frac{1}{\sum_{k=1}^{n} \frac{1}{R_k}} \qquad n = \text{nb de } R$$

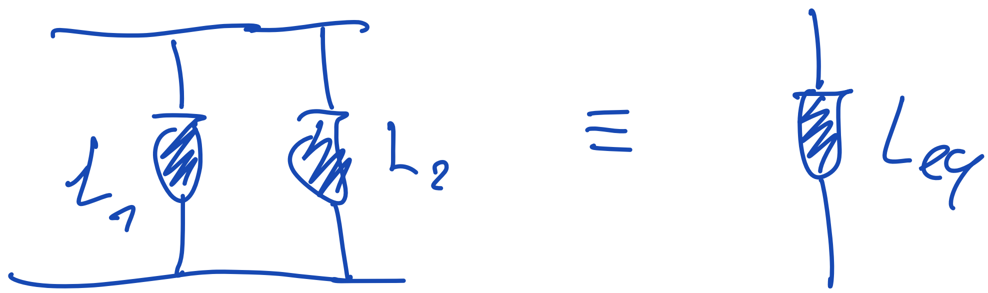
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow R_{eq} = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

5.3.5 Mise en // des C :



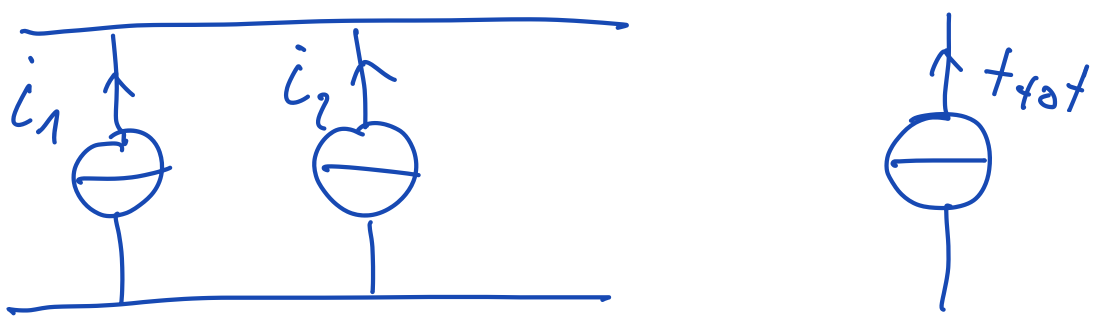
$$// \quad C_{eq} = \sum_{k=1}^{n} C_k \qquad n = \text{nb de } C$$

5.3.6 Mise en // des L



$$// \quad L_{eq} = \frac{1}{\sum_{k=1}^{m} \frac{1}{L_k}} \qquad m = \text{nb de } L$$

5.3.7 Mise en // des sources de courant

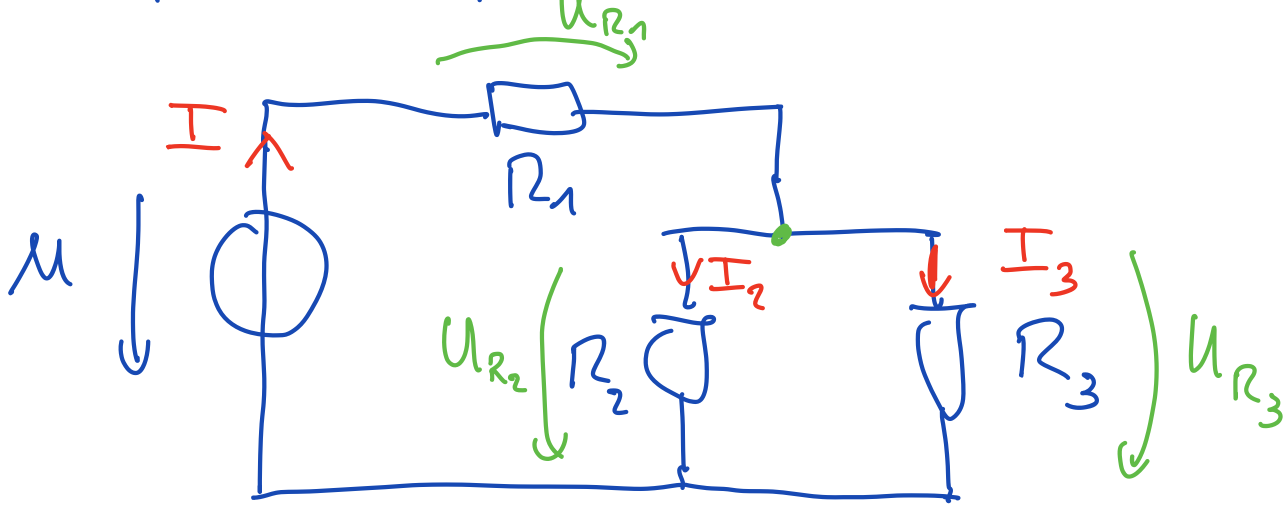


$$i_{tot} = \sum_{k=1}^{m} i_k$$

Mise en // de sources de tension

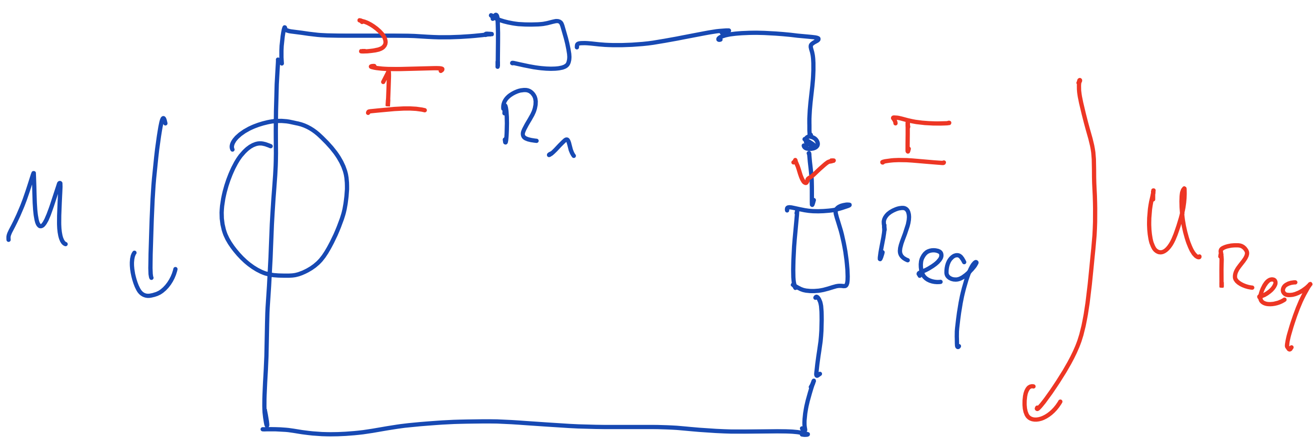
est impossible sauf si toutes les tension ont la même valeur

5.4 Circuits combinés :

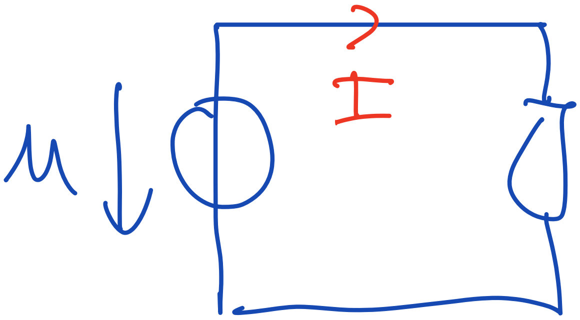


$I = ?$ I_2 ? I_3 ?

$R_2 // R_3$



$$R_{eq} = \frac{R_2 \cdot R_3}{R_2 + R_3}$$



$$R_{tot} = R_1 + R_{eq} = R_1 + \frac{R_2 \cdot R_3}{R_2 + R_3}$$

$$U = R_{tot} \cdot I$$

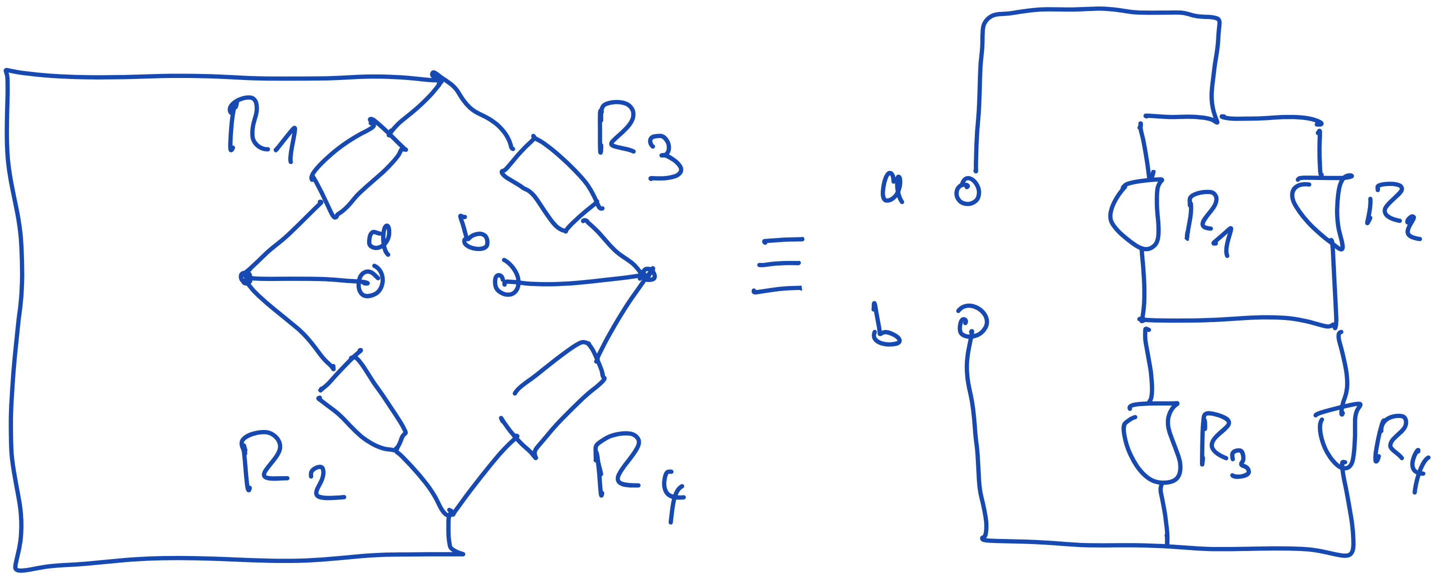
$$I = \frac{U}{R_{tot}}$$

$$U_{R_2} = U_{R_3} = U_{R_{eq}} = R_{eq} \cdot I = R_{eq} \cdot \frac{U}{R_{tot}}$$

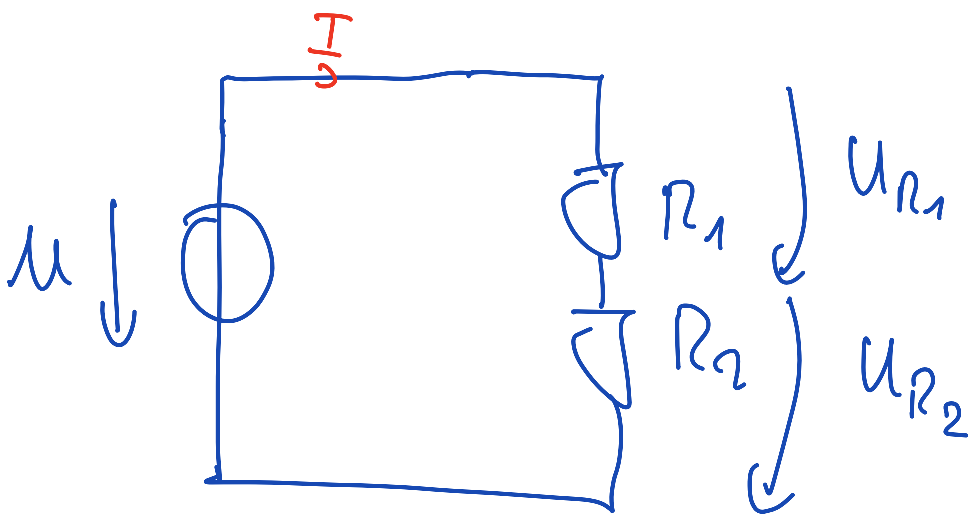
$$I_2 = \frac{U_{R_2}}{R_2} = \frac{U_{R_{eq}}}{R_2}$$

$$I_3 = \frac{U_{R_3}}{R_3} = \frac{U_{R_{eq}}}{R_3}$$

5.4.3 Exemple :



5.5.1 Diviseur de tension :

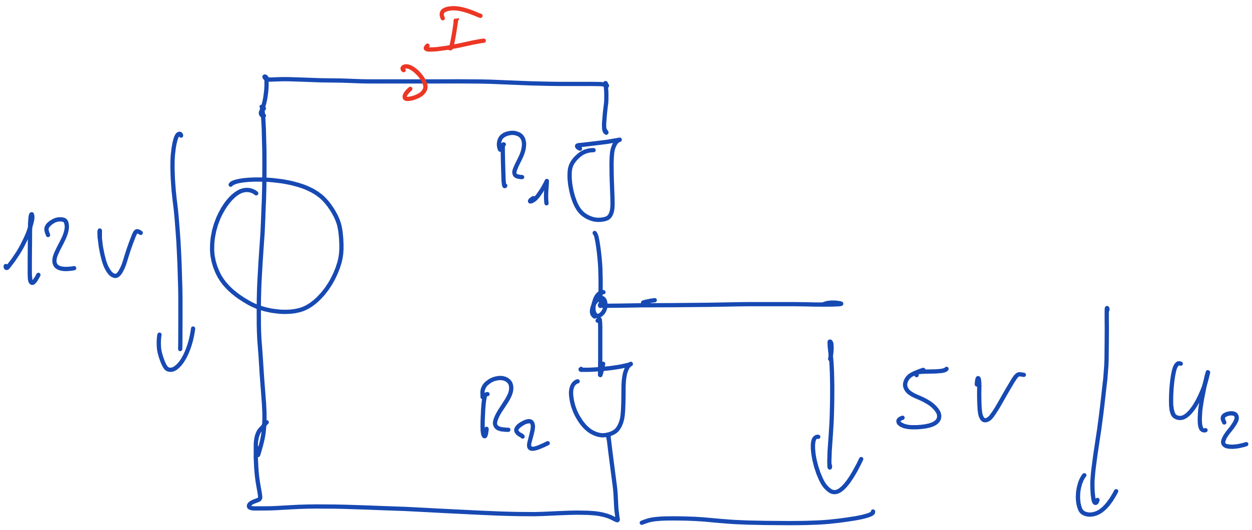


$$U = U_{R_1} + U_{R_2}$$

$$= (R_1 + R_2)\, I$$

$$I = \frac{U}{R_1 + R_2}$$

$$\underline{\underline{U_{R_2}}} = R_2 \cdot I = \underline{\underline{\frac{R_2}{R_1 + R_2} \cdot U}}$$



$$U_2 = \frac{R_1}{R_1 + R_2} \cdot U$$

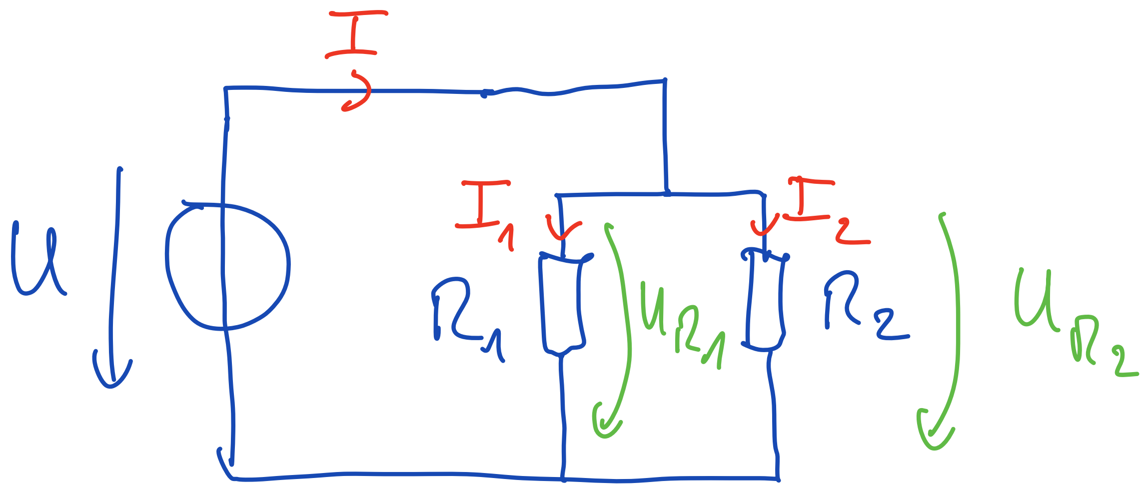
$$5 = \frac{R_2}{R_1 + R_2} \cdot 12$$

$$5 R_1 = 7 R_2$$

$$R_1 = 100\ k\Omega$$

$$R_2 = 71{,}5\ k\Omega$$

5.5.4 Diviseur de courant :



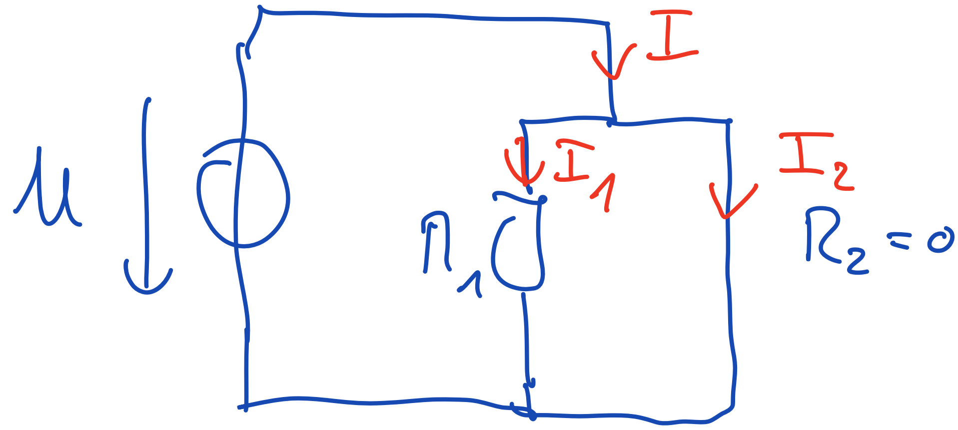
$$U = U_{R_1} = U_{R_2}$$

$$R_{eq} = \frac{R_1 \cdot R_2}{R_1 + R_2} \qquad I = \frac{U}{R_{eq}}$$

$$U_{R_2} = R_2 \cdot I_2 = U = \frac{R_1 \cdot R_2}{R_1 + R_2} \cdot I$$

$$\boxed{I_2 = \frac{R_1}{R_1 + R_2} \cdot I \qquad I_1 = \frac{R_2}{R_1 + R_2} \cdot I}$$

Si :



$$I_2 = \frac{R_1}{R_1 + \cancel{R_2}} \cdot I = I$$

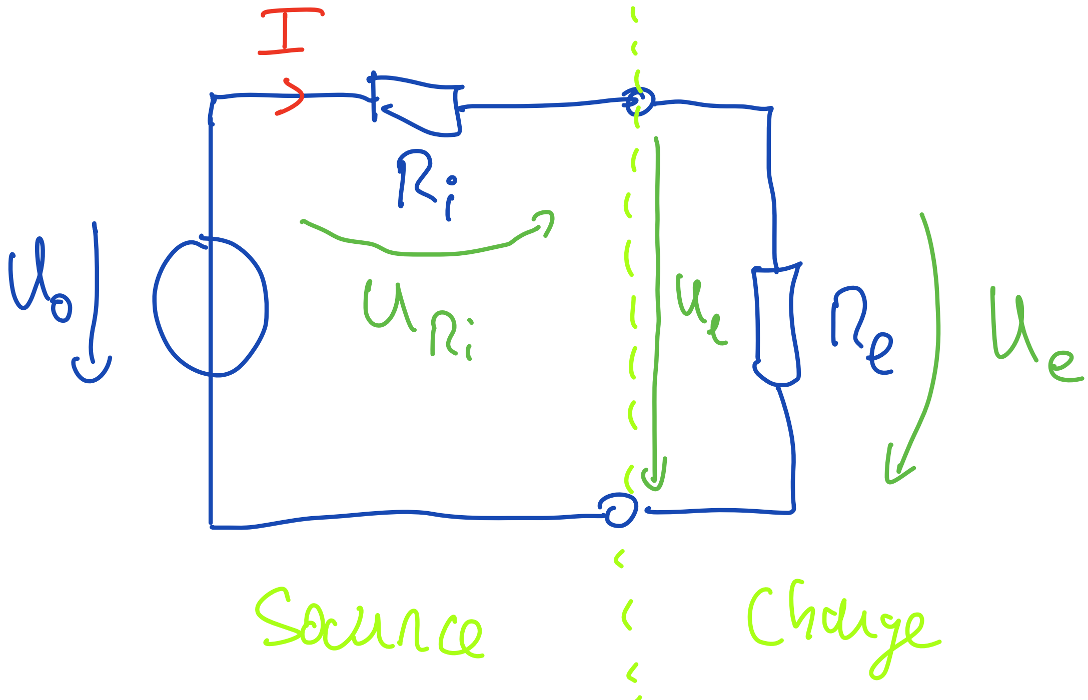
$$I_1 = 0$$

5.6 Méthodes de résolution :

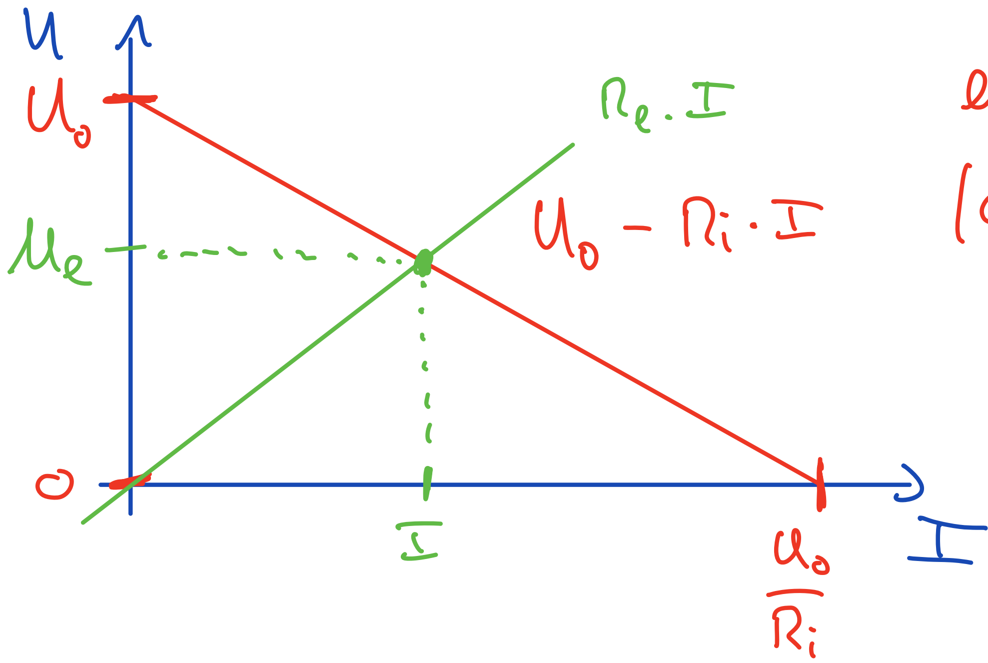
- Redessiner le schéma
- Définir toutes les grandeurs
  U, I, → indice
- Définir le sens des flèches
- Réduire le schéma, série ou //

- Analyse !

5.6.2 Source de tension réelle :



$$U_e = U_0 - U_{R_i} = U_0 - R_i I$$



en court-circuit (cc) : $U_e = 0$

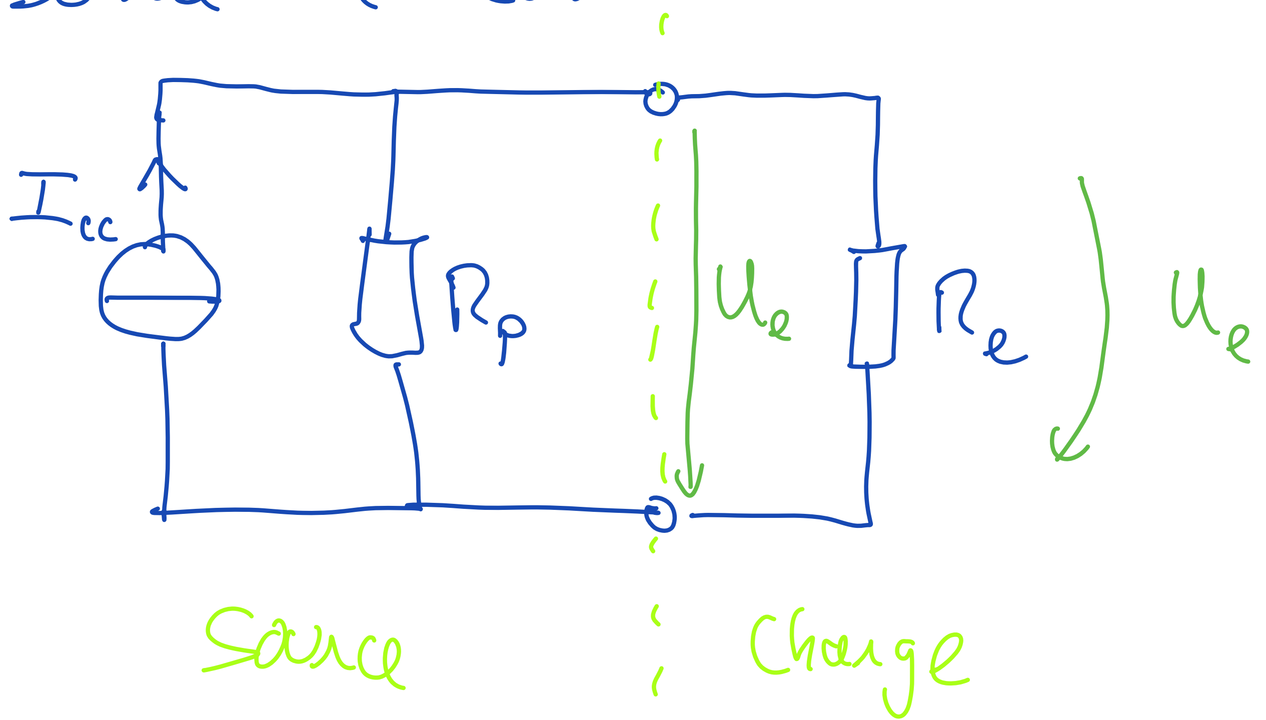
$$I_{cc} = \frac{U_0}{R_i}$$

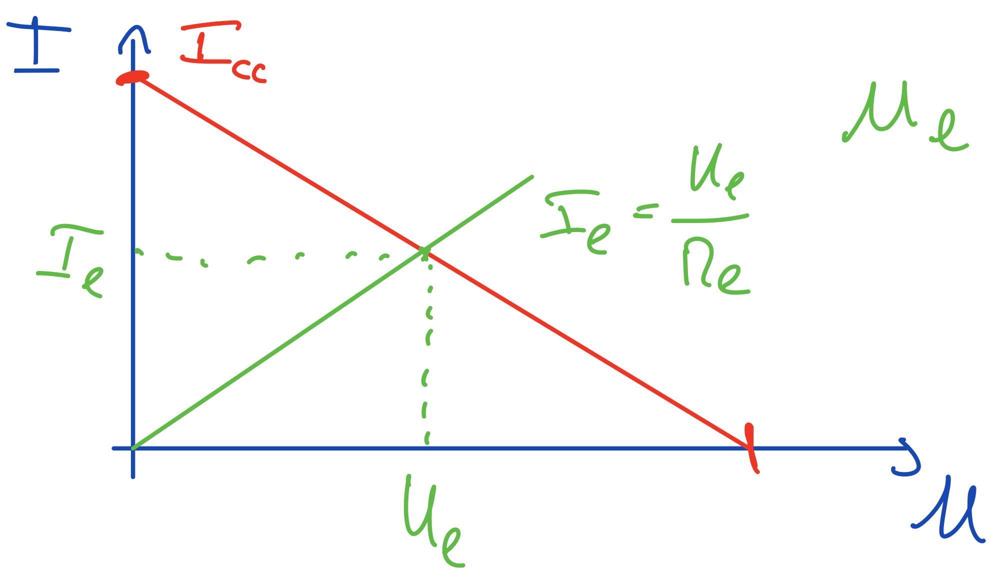
eq de la Charge $\quad U_e = R_e \cdot I$

$$U_e = U_0 \cdot \frac{R_e}{R_e + R_i}$$

$$I_e = \frac{U_0}{R_i + R_e}$$

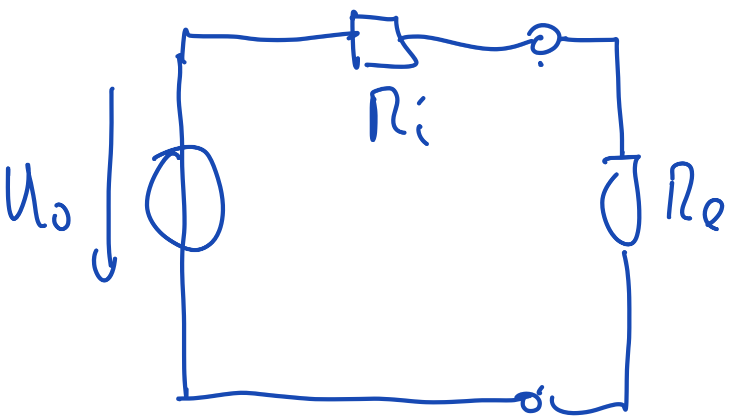
Source de courant réelle :



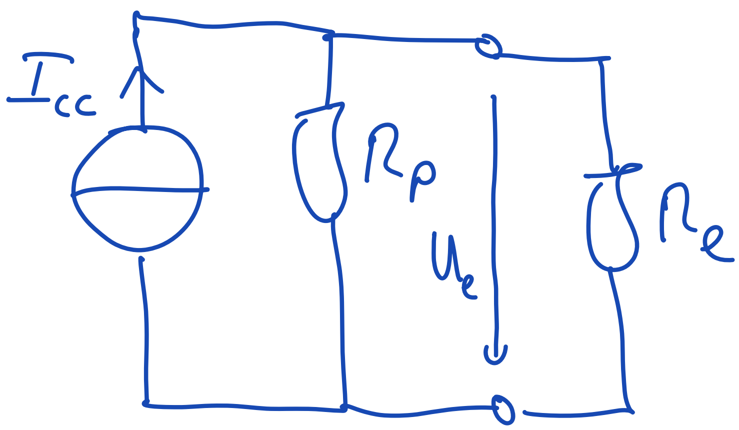


$$U_e = R_e \cdot I_e$$

5.6.3 Equivalence des sources de tension et courant réelles



court-circuit $R_e = 0$	circuit ouvert $R_e \to \infty$
$U_{e_{cc}} = 0$ $I_{e_{cc}} = \frac{U_0}{R_i}$	$U_{e_0} = U_0$
$I_{e_{cc}} = I_{cc}$ $U_{e_{cc}} = 0$	$U_{e_0} = R_p \cdot I_{cc}$

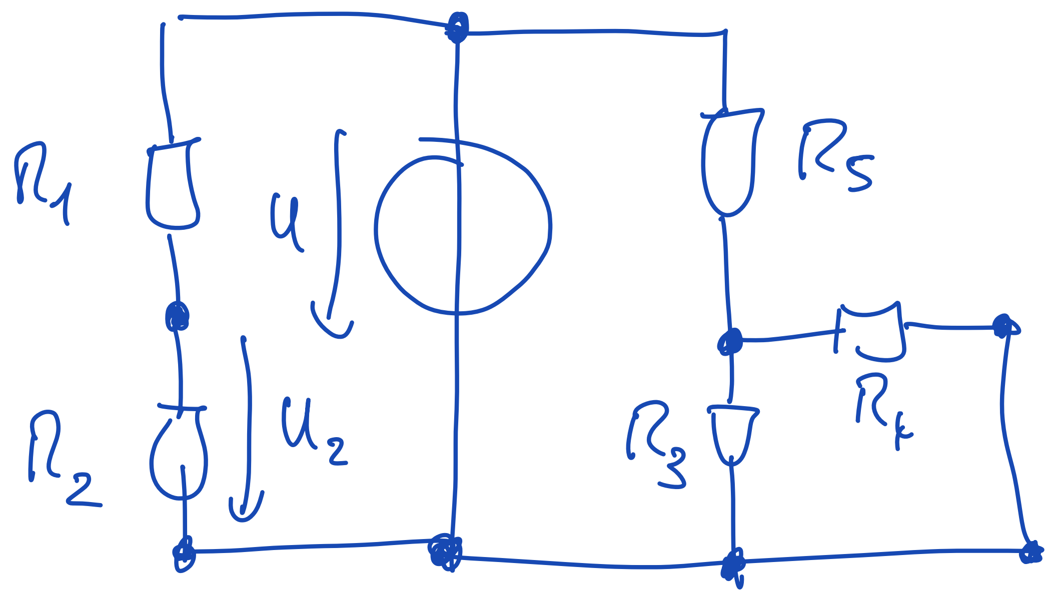


On pose : $U_{e_0} = R_p \cdot I_{cc} = U_0$

$$I_{e_{cc}} = \frac{U_0}{R_i} = I_{cc}$$

$$R_p = \frac{U_0}{I_{cc}} = \frac{U_0}{U_0/R_i} = R_i$$

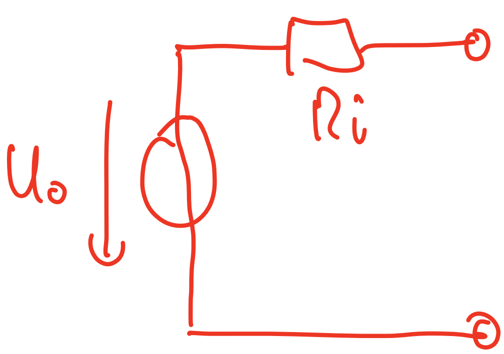
Quiz :



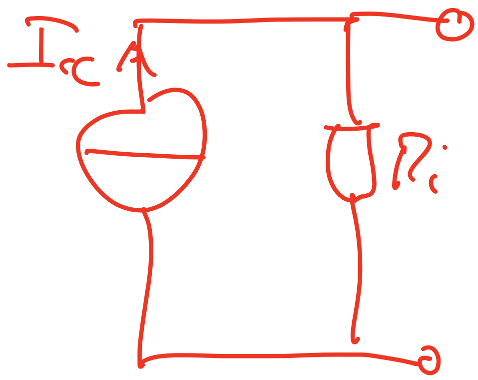
$$U = 12\,V$$

$$U_2 = U \cdot \frac{R_2}{R_1 + R_2} = 5\,V$$

En résumé :

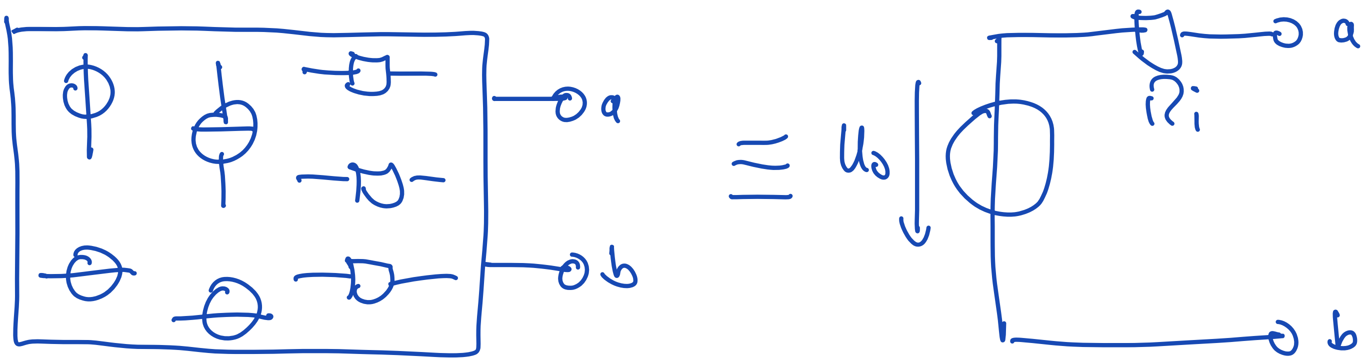


$\equiv$



$$I_{cc} = \frac{U_0}{R_i}$$

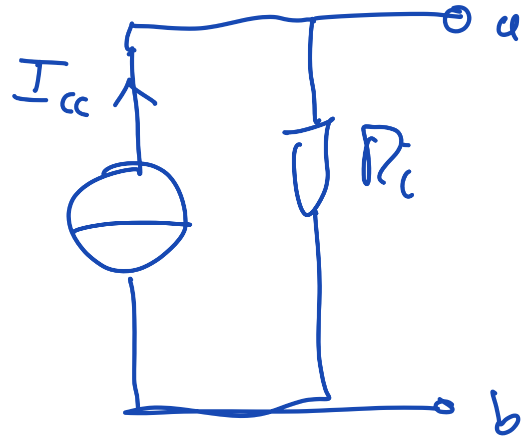
5.7 Théorèmes de Thévenin et Norton :



U_0 = Tension à vide

$$U_0 = U_{ab}\Big|_{I_{ab}=0} \quad (\text{à vide})$$

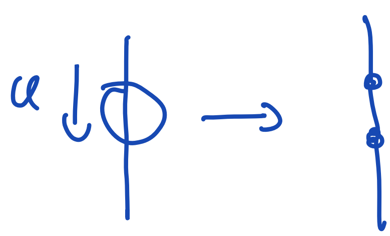
≡

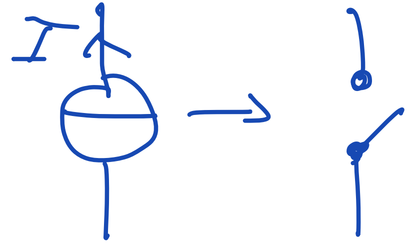


$$I_{cc} = I_{ab}\Big|_{U_{ab}=0} \quad (\text{en court-circuit})$$

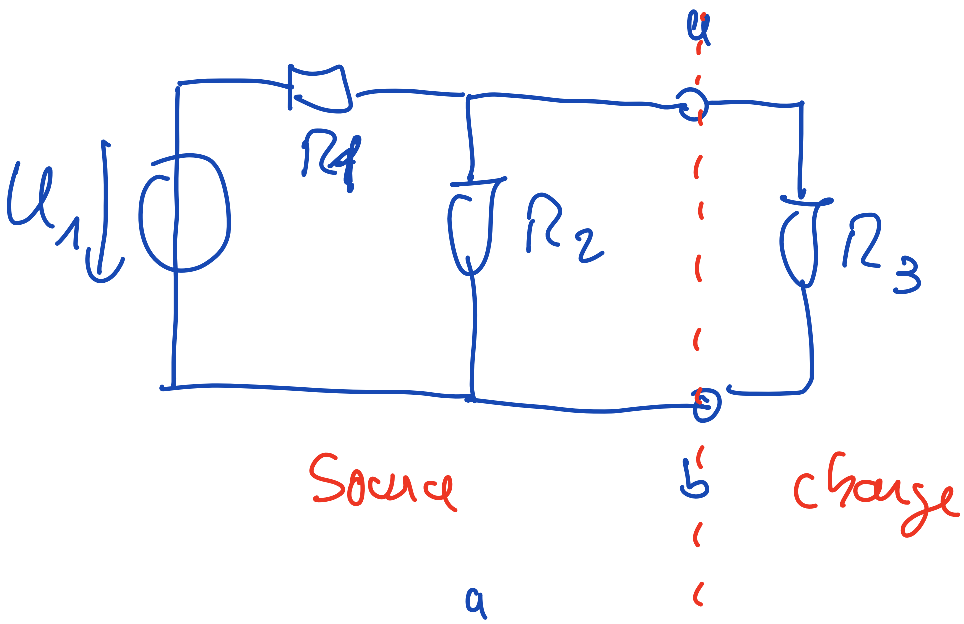
$$R_i = \frac{U_0}{I_{cc}} = R_{ab}\Big|_{\substack{U_j=0 \\ I_j=0}}$$

Annuler une source ⇒

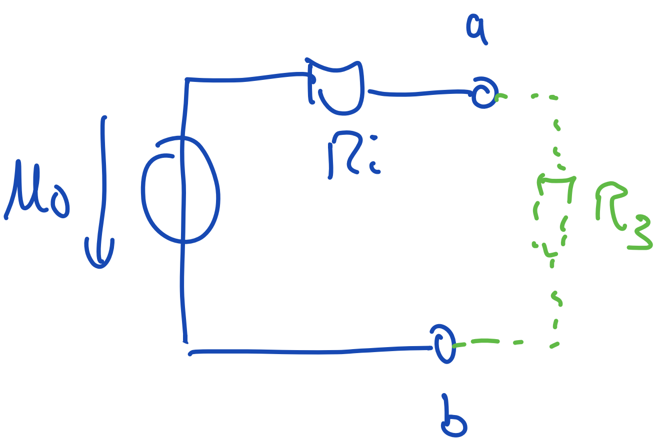




5.7.2 Exemple :

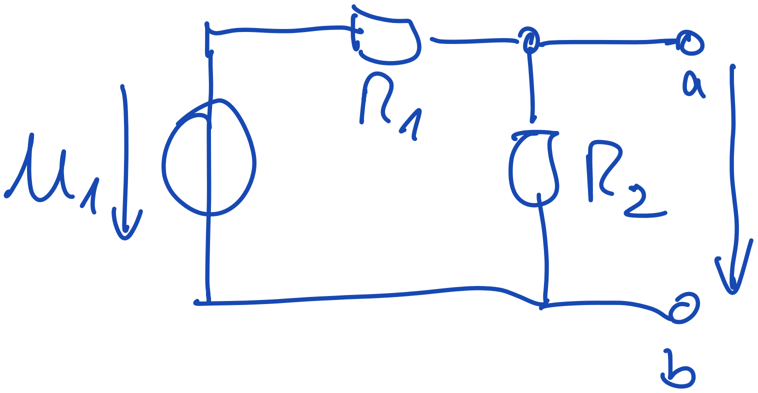


$U_3 = f(R_3)$



U_0 ? Tension à vide entre a et b ?

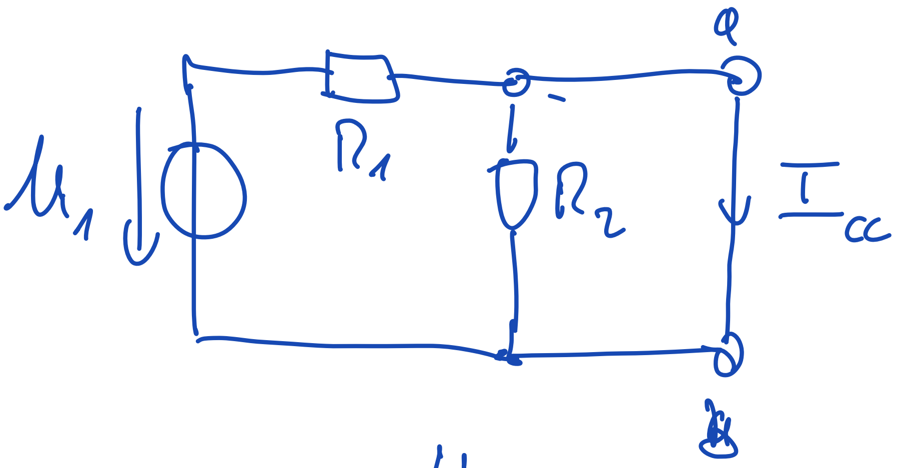
a)



$U_{R_2} = U_0$

$$U_{R_2} = U_0 = U_1 \cdot \frac{R_2}{R_1 + R_2}$$

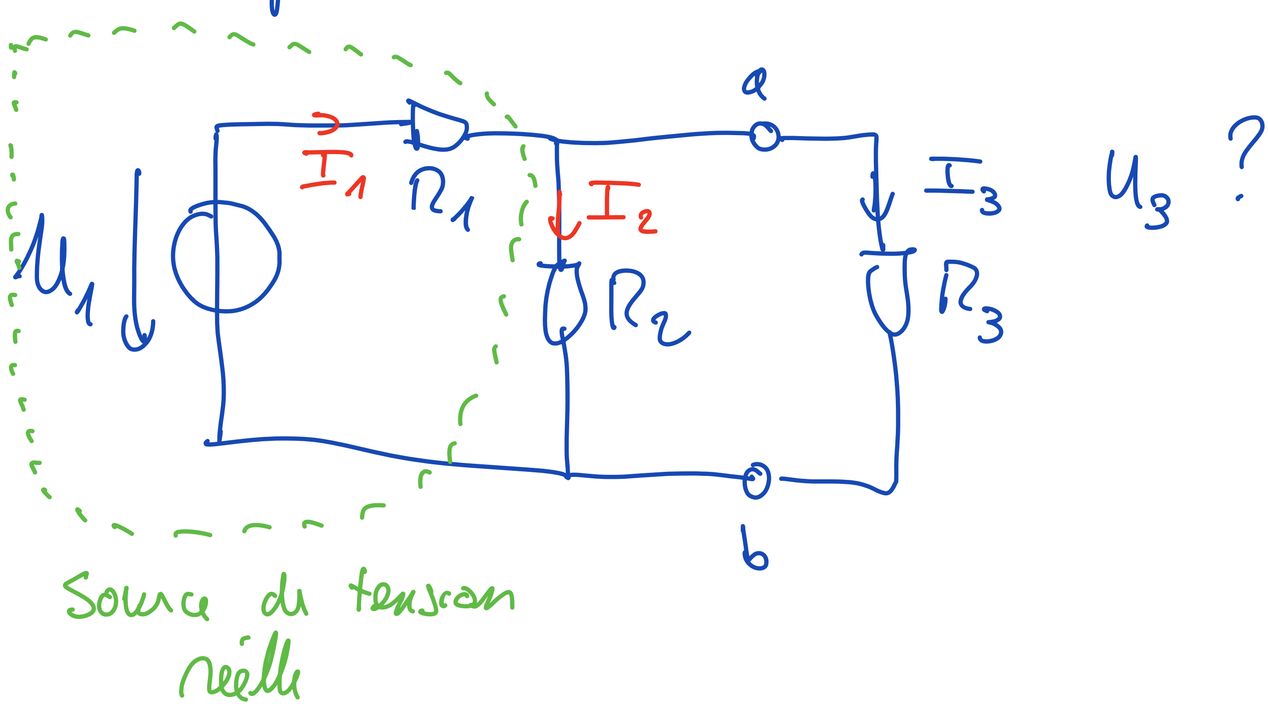
b) I_{cc} = courant de court circuit

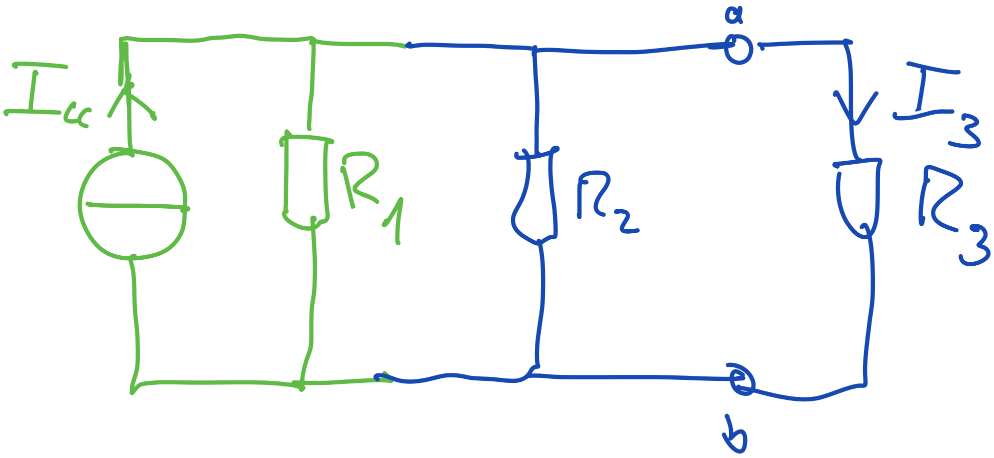


$$I_{cc} = \frac{U_1}{R_1}$$

c) $$R_i = \frac{U_0}{I_{cc}} = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

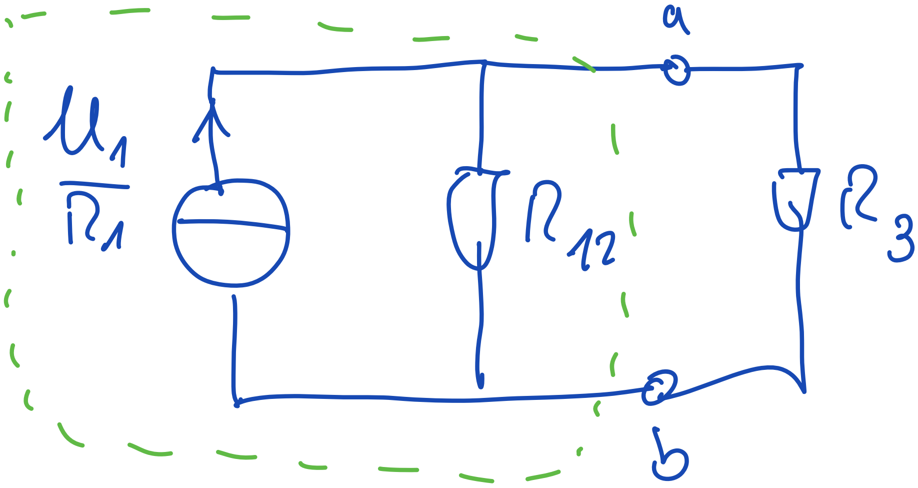
Autre possibilité :



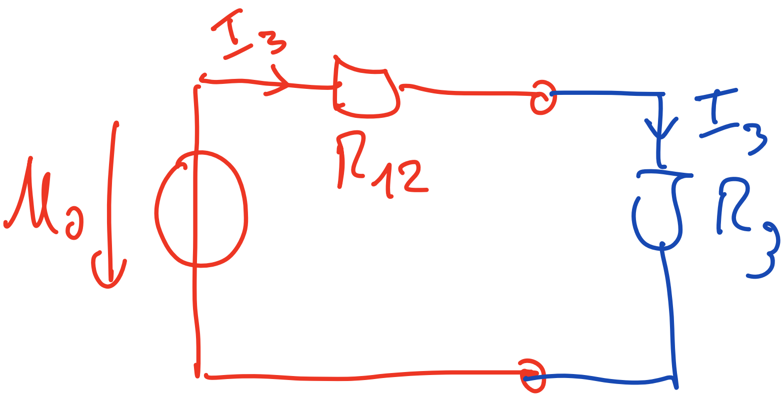


$$I_{cc} = \frac{U_1}{R_1}$$

$$R_{12} = \frac{R_1 \cdot R_2}{R_1 + R_2}$$



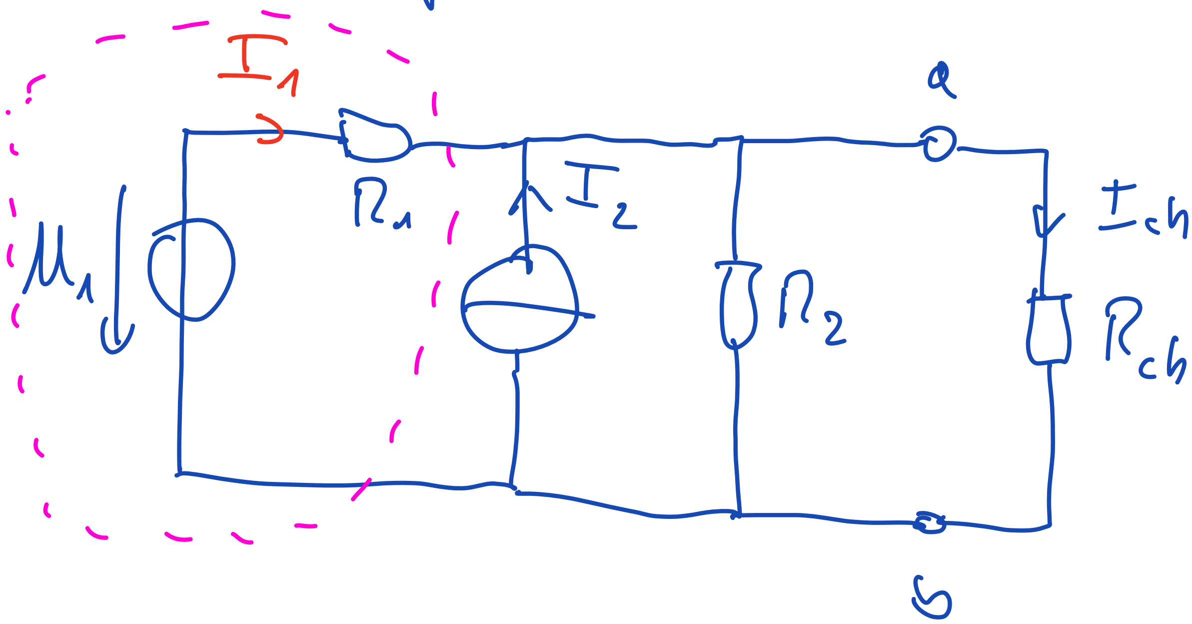
Source de courant réelle

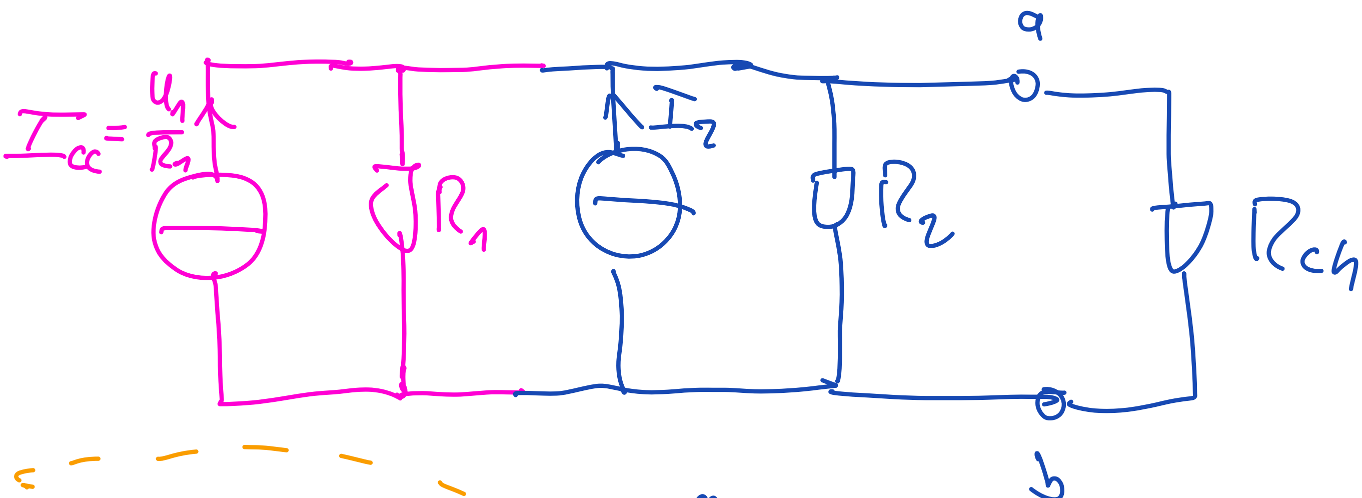


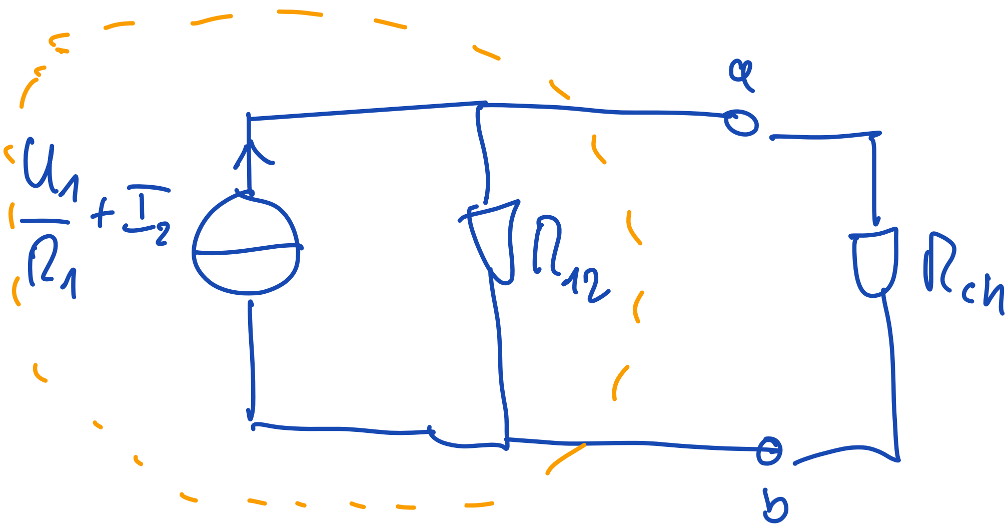
$$U_0 = R_{12} \cdot \frac{U_1}{R_1}$$

$$I_3 = \frac{U_0}{R_{12} + R_3}$$

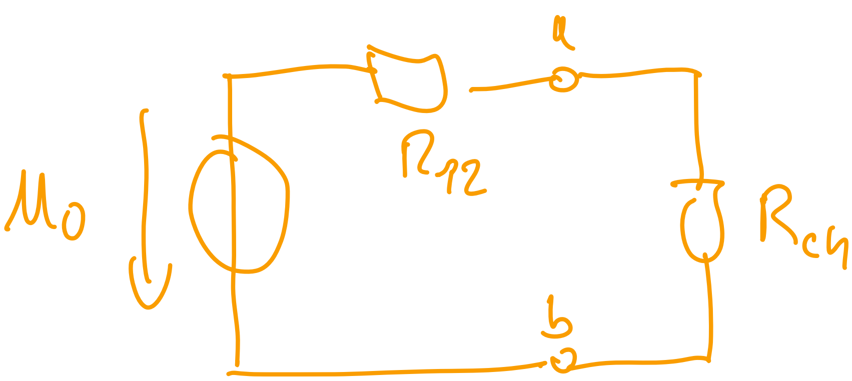
Autre exemple :







$$R_{12} = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

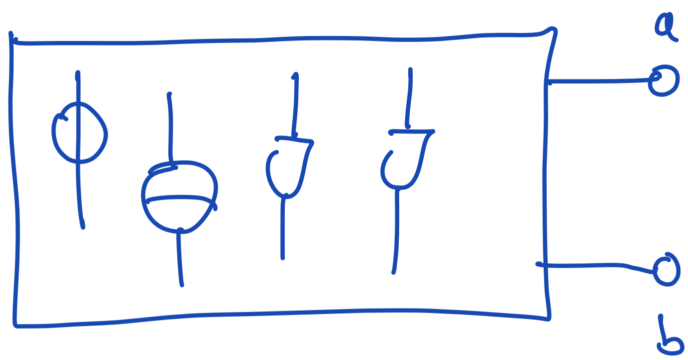


$$U_0 = R_{12}\left(\frac{U_1}{R_1} + I_2\right)$$

$$I_{ch} = \frac{U_0}{R_{12} + R_{ch}}$$

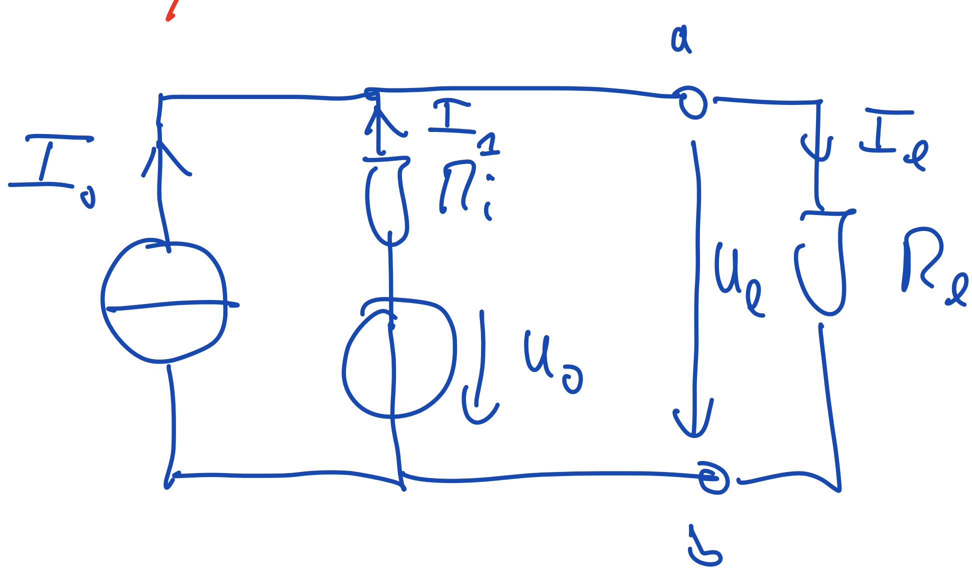
5.8 Principe de superposition

Définition :



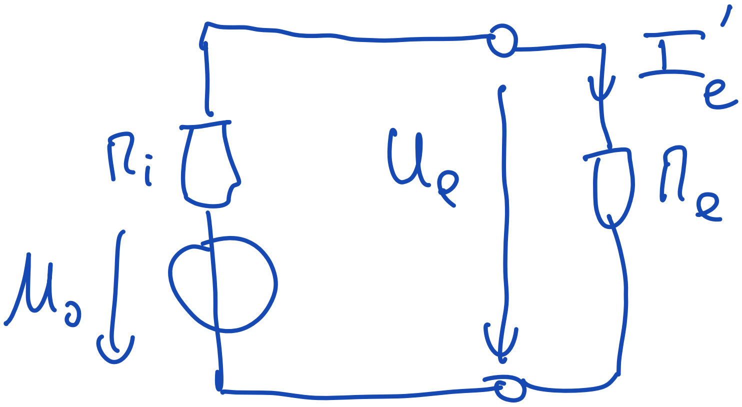
L'action résultante est la somme algébrique des actions séparées de chaque source, les autres étant annulées

Le système doit être linéaire !!



I_e, u_e ?

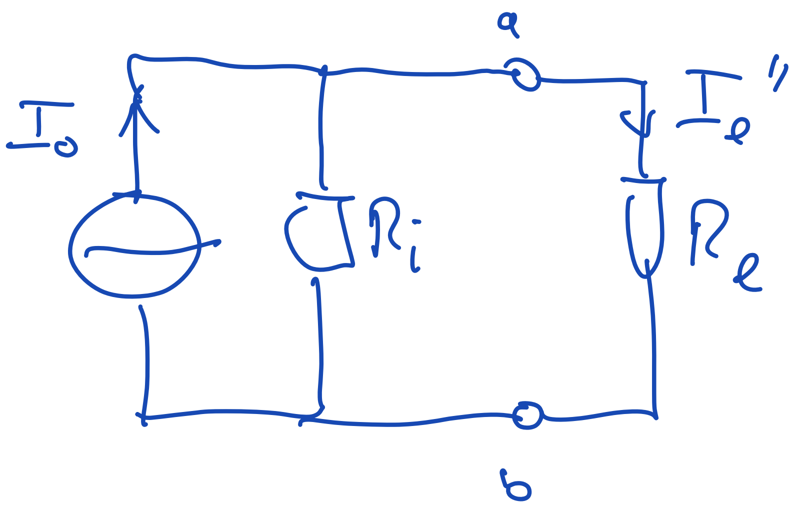
1) On annule la source de courant :



$$I_e' = \frac{U_0}{R_i + R_e}$$

$$U_e' = U_0 \cdot \frac{R_e}{R_e + R_i}$$

2) On annule la source de tension :



$$I_e'' = I_0 \cdot \frac{R_i}{R_i + R_e}$$

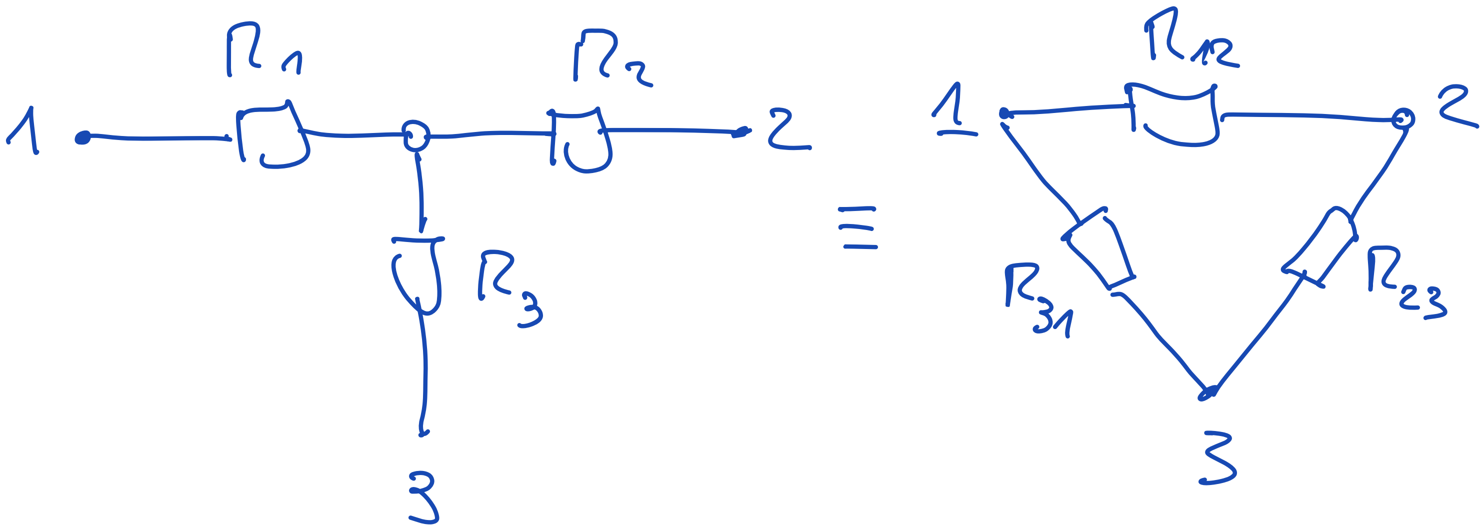
$$U_e'' = R_e \cdot I_e'' = I_0 \cdot \frac{R_e \cdot R_i}{R_e + R_i}$$

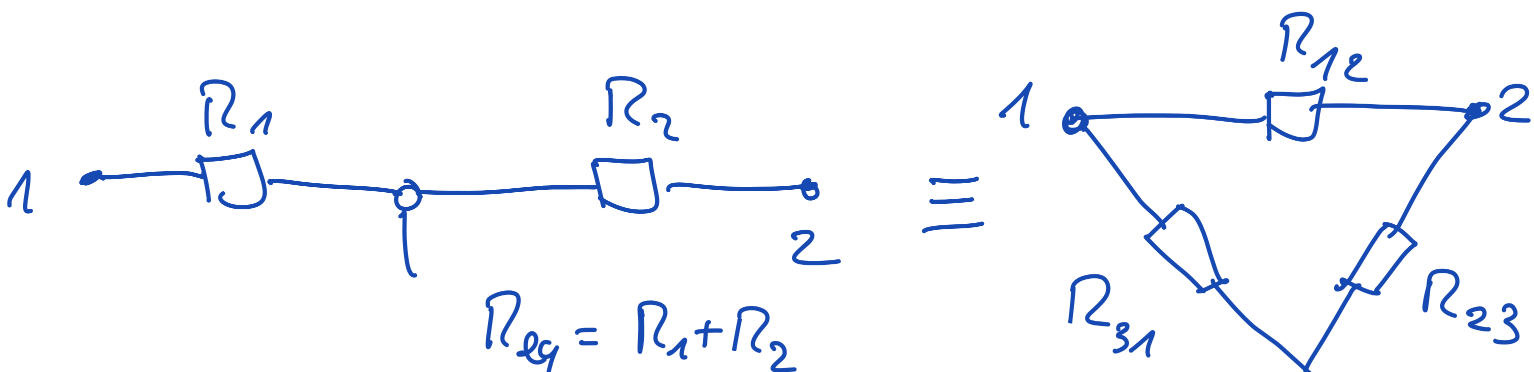
$$I_e = I_e' + I_e''$$

$$U_e = U_e' + U_e''$$

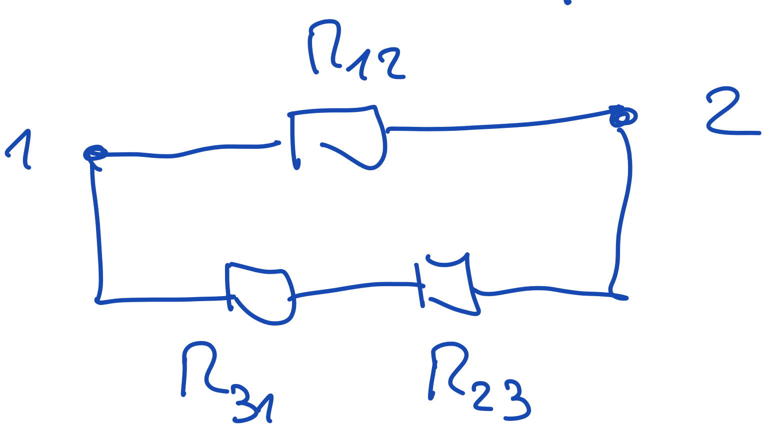
5.9 Transformation Etoile - Triangle

il s'agit d'un tripole :





$$R_{eq} = R_1 + R_2$$



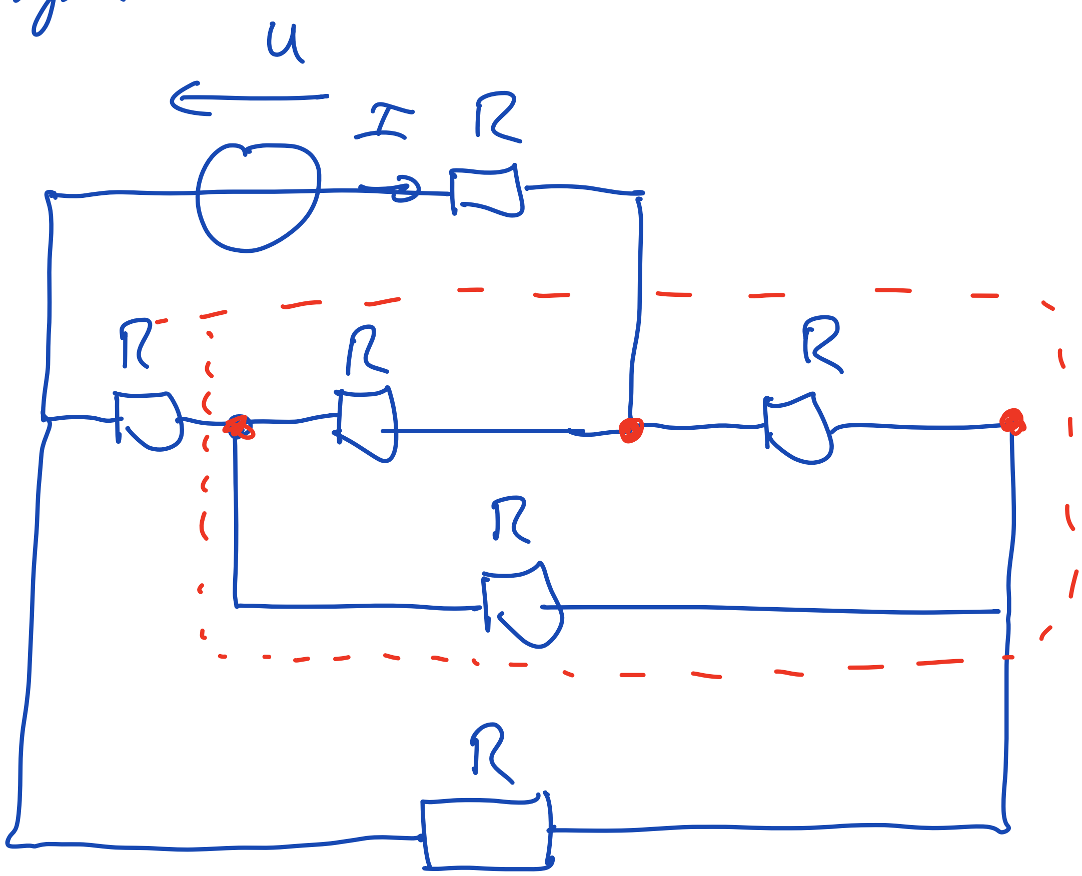
$$R_{eq} = \frac{R_{12} \cdot (R_{31} + R_{23})}{R_{12} + R_{31} + R_{23}}$$

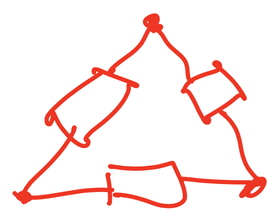
$$R_1 = \frac{R_{31} \cdot R_{12}}{R_{12} + R_{23} + R_{31}} \qquad R_{12} = R_1 + R_2 + \frac{R_1 \cdot R_2}{R_3}$$

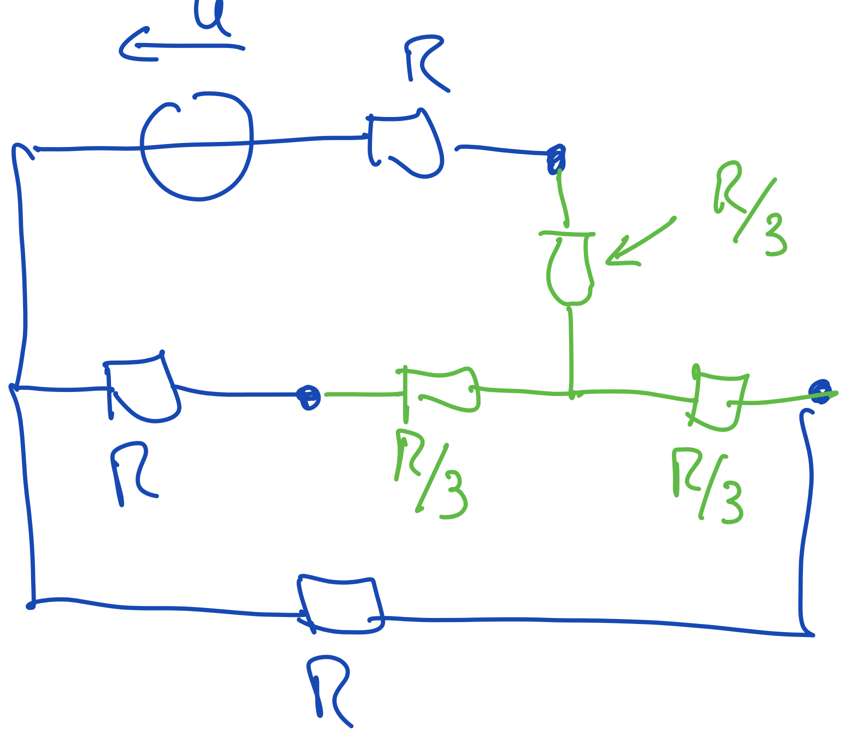
$$R_2 = \frac{R_{12} \cdot R_{23}}{R_{12} + R_{23} + R_{31}} \qquad R_{23} = R_2 + R_3 + \frac{R_2 \cdot R_3}{R_1}$$

$$R_3 = \frac{R_{23} \cdot R_{31}}{R_{12} + R_{23} + R_{31}} \qquad R_{31} = R_3 + R_1 + \frac{R_3 \cdot R_1}{R_2}$$

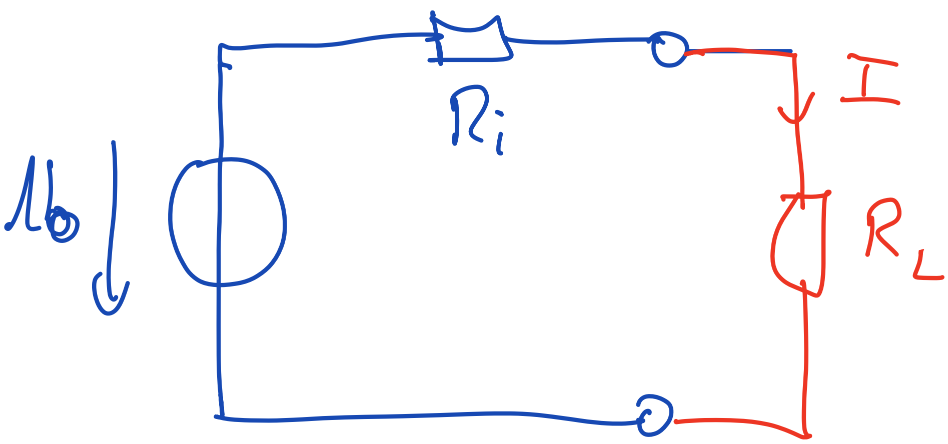
Exemple :







5.11 Puissance maximum transmise par un dipôle :



Puissance transmise à $R_L \to$ max

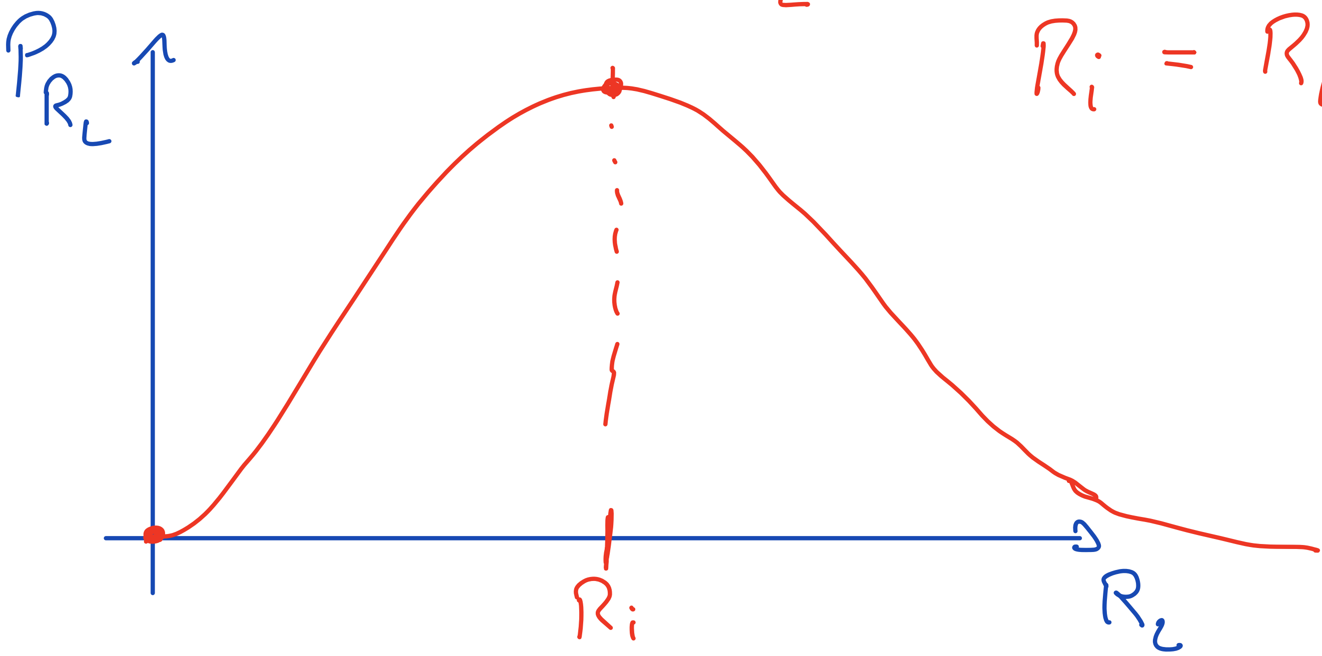
$$P_{R_L} = R_L \cdot I^2 = U_{R_L} \cdot I$$

$$I = \frac{U_0}{R_i + R_L}$$

$$P_{R_L} = R_L \frac{U_0^{\,2}}{(R_i + R_L)^2}$$

Que doit valoir R_L par rapport à R_i pour P_{R_L} max

Max : $\longrightarrow \dfrac{d P_{R_L}}{d R_L} = 0$



$R_i = R_L$

$\longrightarrow$ l'adaptation de puissance

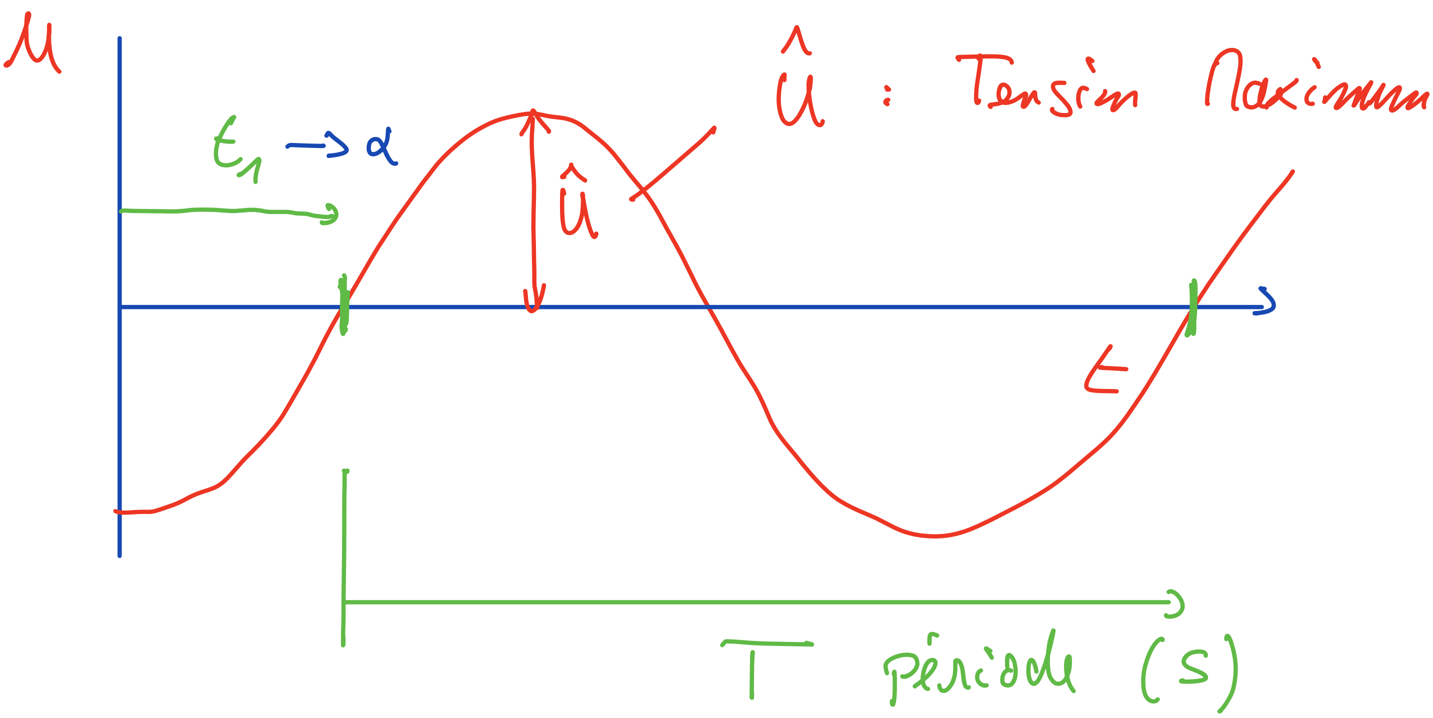
$$\eta = \frac{P_{utile}}{P_{consommée}} = \frac{R_L \cdot \cancel{I^2}}{(R_L + R_i)\cancel{I^2}}$$

Si $R_i = R_L$

$\Rightarrow \eta = 0,5$

6. Régime sinusoïdal Monophasé :

6.2 Grandeurs Sinusoïdales



T = période du signal (s)

f = fréquence $= \dfrac{1}{T}$ (Hz)

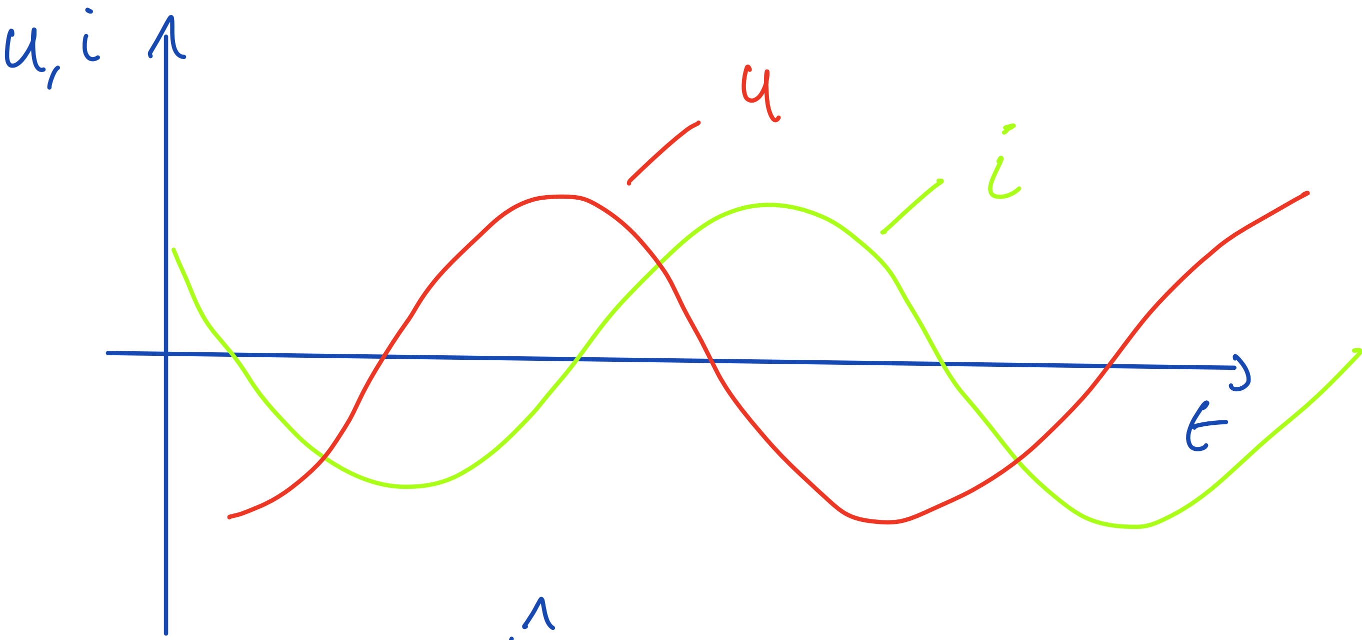
ω = pulsation $= \dfrac{2\pi}{T} = 2\pi \cdot f$

$$\alpha = \frac{t_1 \cdot 2\pi}{T}$$

$$u(t) = \hat{U} \sin(\omega t + \alpha)$$

$$u = \hat{U} \sin(\omega t + \alpha$$

minuscule : → grandeur instantanée



$$u = \hat{U} \sin(\omega t + \alpha)$$

$$i = \hat{I} \sin(\omega t + \beta)$$

Définition : $\varphi = \alpha - \beta$

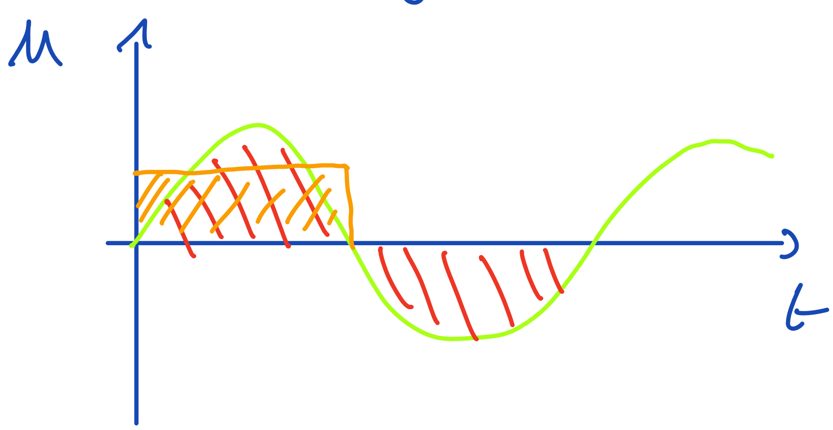
déphasage entre u et i

Définition de la valeur moyenne :

$$\bar{x} = \frac{1}{T}\int_0^T x(t)\,dt$$

$$\bar{u} = \frac{1}{T}\int_0^T u\,dt$$

$$\bar{u} = \frac{1}{T}\int_0^T \hat{U}\sin(\omega t+\alpha)\,dt$$



Moyenne sur $T/2$:

$T/2$

$$\bar{u}\Big|_{T/2} = \frac{1}{T/2}\int_0^{T/2} \hat{u}\sin(\omega t)\,dt \qquad \alpha = 0$$

$$\bar{u}\Big|_{T/2} = \frac{1}{\omega}\,\frac{2\hat{u}}{T}\left[-\cos\left(\omega\cdot\frac{T}{2}\right) + \cos(0)\right]$$

$$\omega = \frac{2\pi}{T}$$

$$\frac{\cancel{2}\,\hat{u}\,\cancel{T}}{\cancel{T}\,\cancel{2}\cdot\pi}\left[-\cos\left(\frac{\cancel{T}}{\cancel{2}}\,\frac{\cancel{2}\pi}{\cancel{T}}\right) + \cos(0)\right]$$

$$= \frac{\hat{u}}{\pi}\left[1 + 1\right] = \frac{2}{\pi}\hat{u}$$

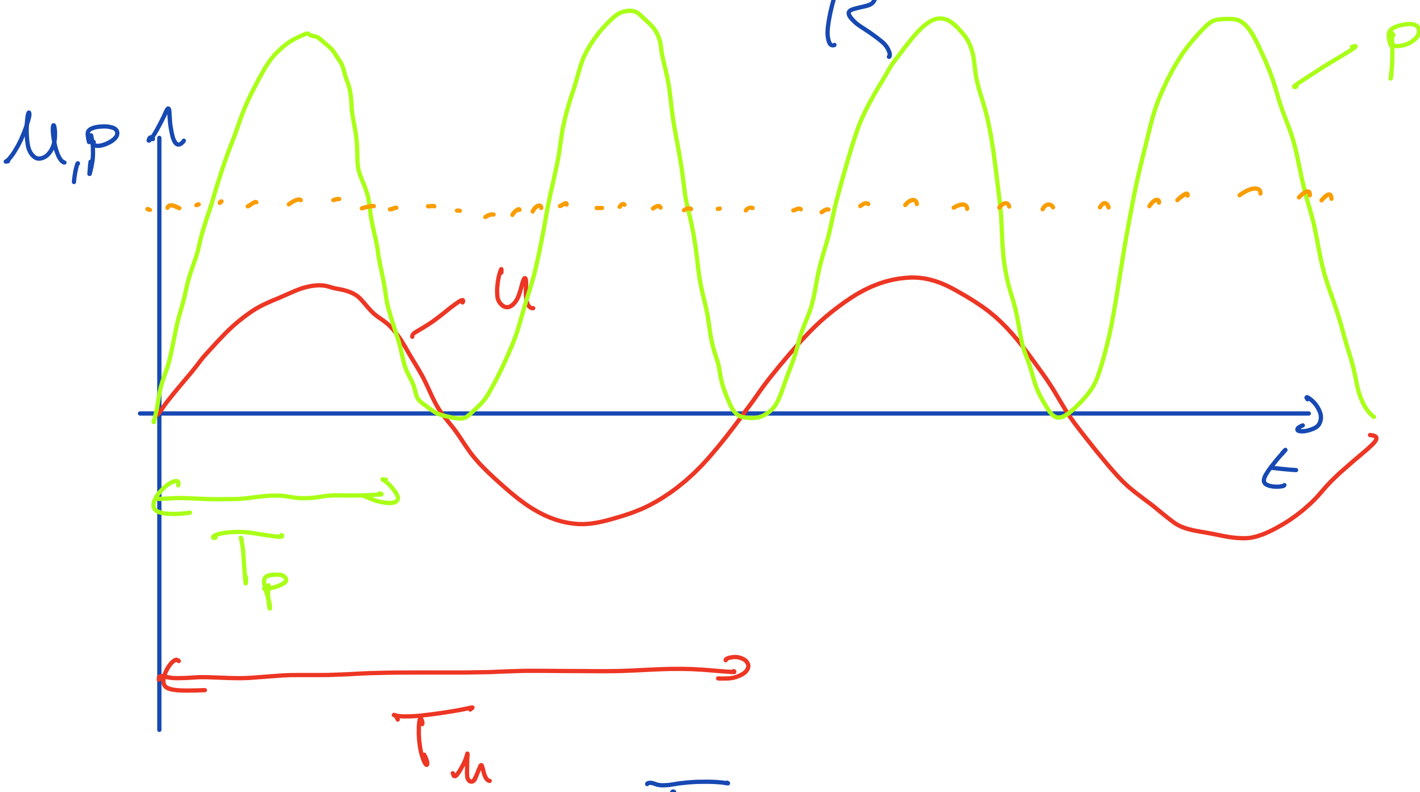
6.2.13 Puissance instantanée

$$p = u \cdot i$$

$$p = R \cdot i^2 = \frac{u^2}{R}$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$= \frac{\hat{u}^2 \sin^2(\omega t + \alpha)}{R}$$



$$\bar{P}_R = \frac{1}{T} \int_0^T \frac{\hat{u}^2}{R} \sin^2(\omega t + \alpha)\, dt$$

$$\bar{P}_R = \frac{1}{T} \int_0^T \frac{\hat{u}^2}{R} \cos^2(\omega t + \alpha)\, dt$$

$$2\,\overline{P}_R = \frac{1}{T}\int_0^T \frac{\hat{u}^2}{R}\cdot 1\, dt$$

$$2\,\overline{P}_R = \frac{\hat{u}^2}{R} \rightarrow \overline{P}_R = \frac{\hat{u}^2}{2R}$$

En alternatif : $\overline{P}_R = \frac{\hat{u}^2}{2R}$

En continu : $P_R = \frac{u^2}{R}$

6.2.12 Valeur efficace :

(RMS Root Mean . Square)

Def : $U = \sqrt{\frac{1}{T}\int_0^T \hat{u}^2 \sin^2(\omega t+\alpha)\, dt}$

Majuscule

$\sin^2 x$

$$= \frac{1-\cos 2\alpha}{2}$$

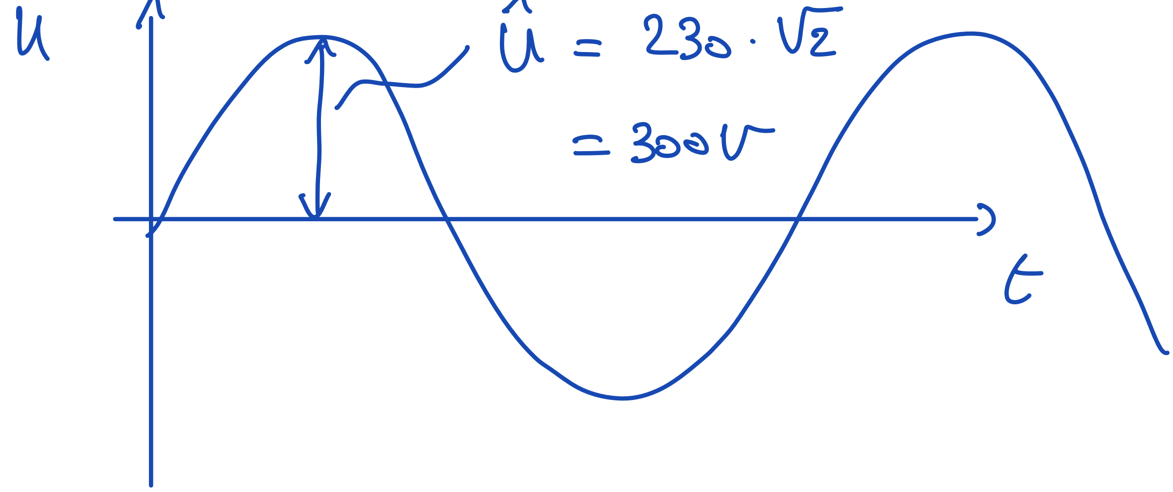
$$= \sqrt{\frac{\hat{u}^2}{T}\int_0^T \frac{1}{2}\,dt - \frac{1}{2}\frac{\hat{u}^2}{T}\underbrace{\int_0^T \cos(2\omega t - 2\alpha)\,dt}_{0}}$$

$$\boxed{u = \frac{\hat{u}}{\sqrt{2}}}$$

$$\hat{u} = u \cdot \sqrt{2}$$

crête — efficace — coef. avec sinus

$$\bar{P}_R = \frac{\hat{u}^2}{2R} = \frac{u^2}{R}$$



u, i valeurs instantanés

U, I valeurs efficaces

$\hat{U}, \hat{I}$ valeurs crêtes

$\bar{u}, \bar{i}$ valeurs moyennes

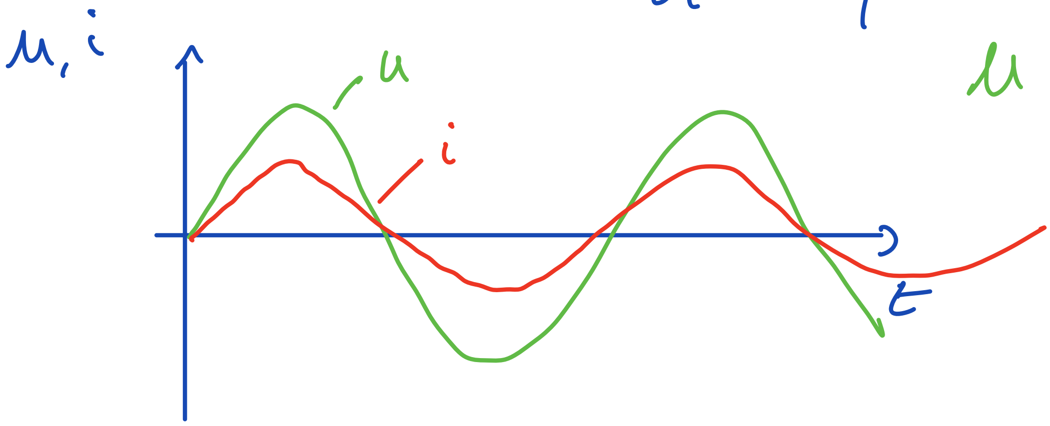
6.2.14 Cas de R

$$u = R \cdot i$$

$$\hat{U} \cos(\omega t + \alpha) = R \cdot \hat{I} \cos(\omega t + \beta)$$

Donc : $\hat{U} = R \cdot \hat{I}$

$$\alpha = \beta$$



u est en phase avec i

6.2.15 cas de L

$$u = L \frac{di}{dt}$$

$$\hat{U} \cos(\omega t + \alpha) = -\omega L \hat{I} \sin(\omega t + \beta)$$

$$= \omega L \hat{I} \cos\left(\omega t + \beta + \frac{\pi}{2}\right)$$

$$\hat{U} = \omega L \hat{I}$$

$$\alpha = \beta + \frac{\pi}{2}$$

Tension et le courant sont quadrature

retard du courant de $\frac{\pi}{2}$ sur la tension.

6.2.16 Cas de C

$$i = C \frac{du}{dt}$$

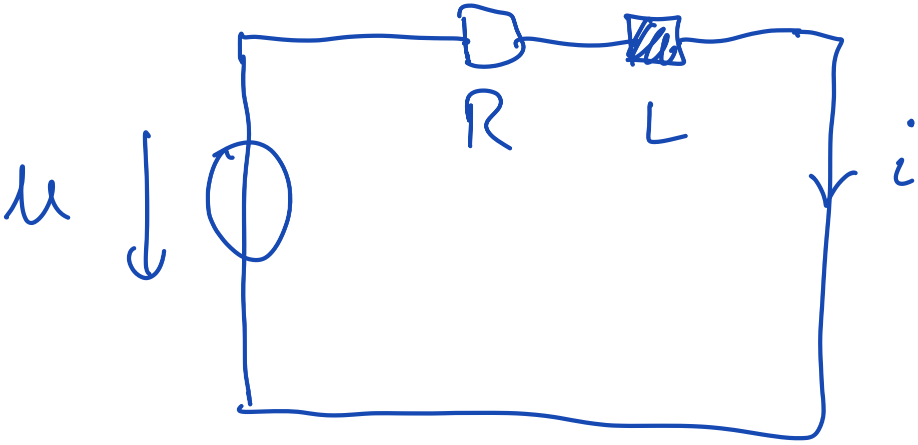
$$\hat{I} \cos(\omega t + \beta) = -\omega C \hat{U} \sin(\omega t + \alpha)$$
$$= \omega C \hat{U} \cos\left(\omega t + \alpha + \frac{\pi}{2}\right)$$

$$\hat{U} = \frac{\hat{I}}{\omega C}$$

$$\alpha = \beta - \frac{\pi}{2}$$

courant en avance de $\pi/2$ sur la tension

6.3 Calcul complexe associé :



$$u = u_R + u_L$$

$$u = \hat{U} \sin(\omega t + \alpha)$$ connu

$$i = \hat{I} \sin(\omega t + \beta)$$ inconnu

$\longrightarrow$ simplification : $\alpha = 0$

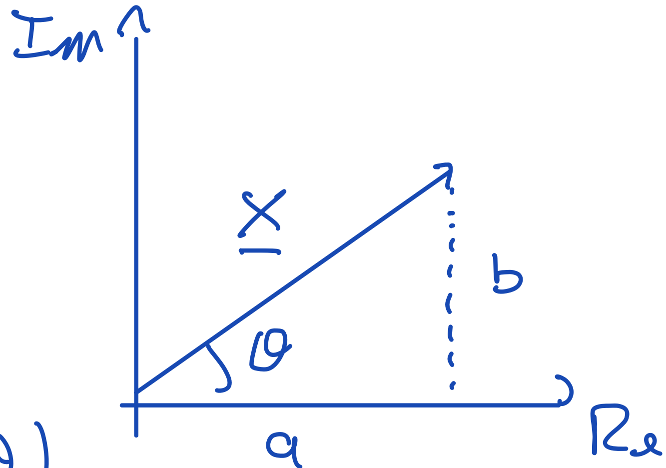
$$\hat{U} \sin(\omega t) = R \cdot \hat{I} \sin(\omega t + \beta) + \omega L \hat{I} \cos(\omega t + \beta)$$

$\vdots$

compliqué !!

Rappel : $j = \sqrt{-1}$

$$\underline{X} = a + bj$$



$$= \hat{X}(\cos\theta + j\sin\theta)$$

$$= \hat{X} e^{j\theta}$$

Concept :

$$u = \hat{U}\sin(\omega t) \xrightarrow{\text{Trans. compl.}} \underline{u} = \hat{U} e^{j(\omega t)}$$

Reel — imaginaire

$$\xleftarrow{u = \mathrm{Im}\{\underline{u}\}}$$

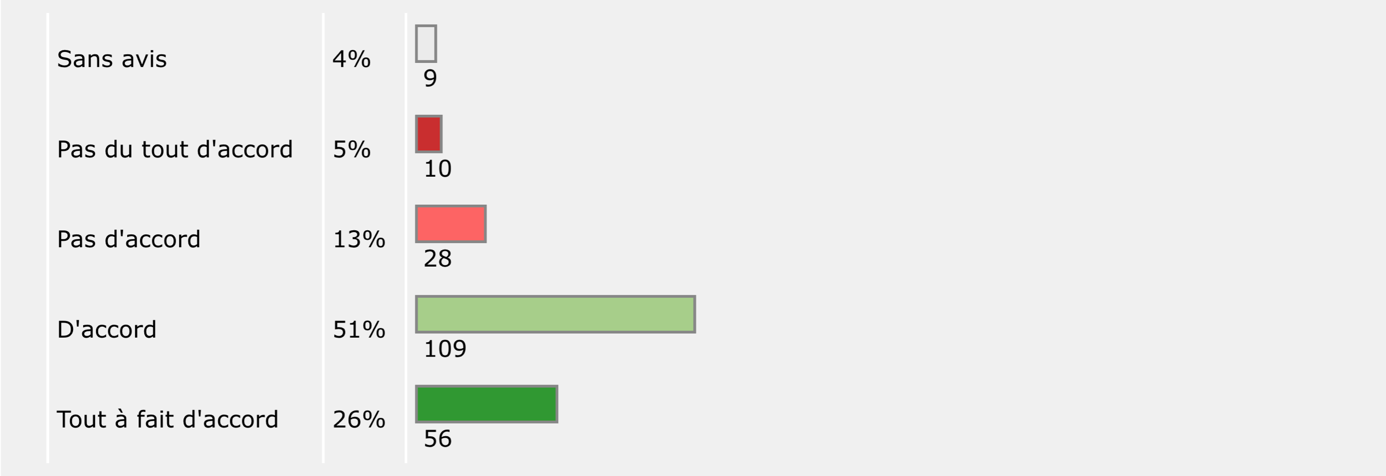
$$i = \hat{I}\sin(\omega t+\beta) \longrightarrow \underline{i} = \hat{I} e^{j(\omega t+\beta)}$$

$$\longleftarrow$$

$$i = \mathrm{Im}\{\underline{i}\}$$

Année	2024-2025
Matière	Electrotechnique I
Questionnaire	Retour indicatif des enseignements (dès 2022-2023)
Nb Inscrit	309
Nb Répondu	212

Le déroulement du cours permet ma formation et un climat de classe approprié



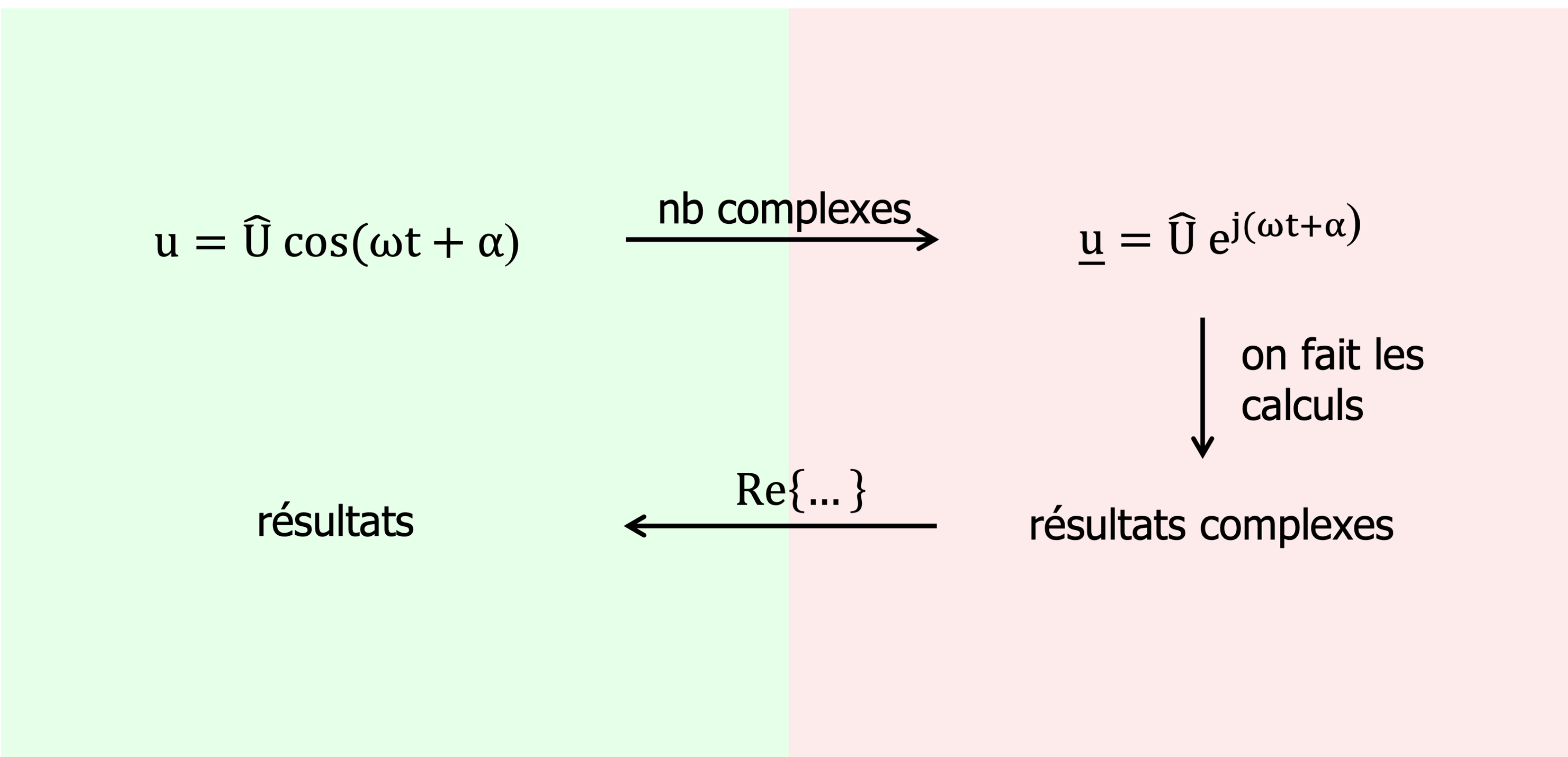
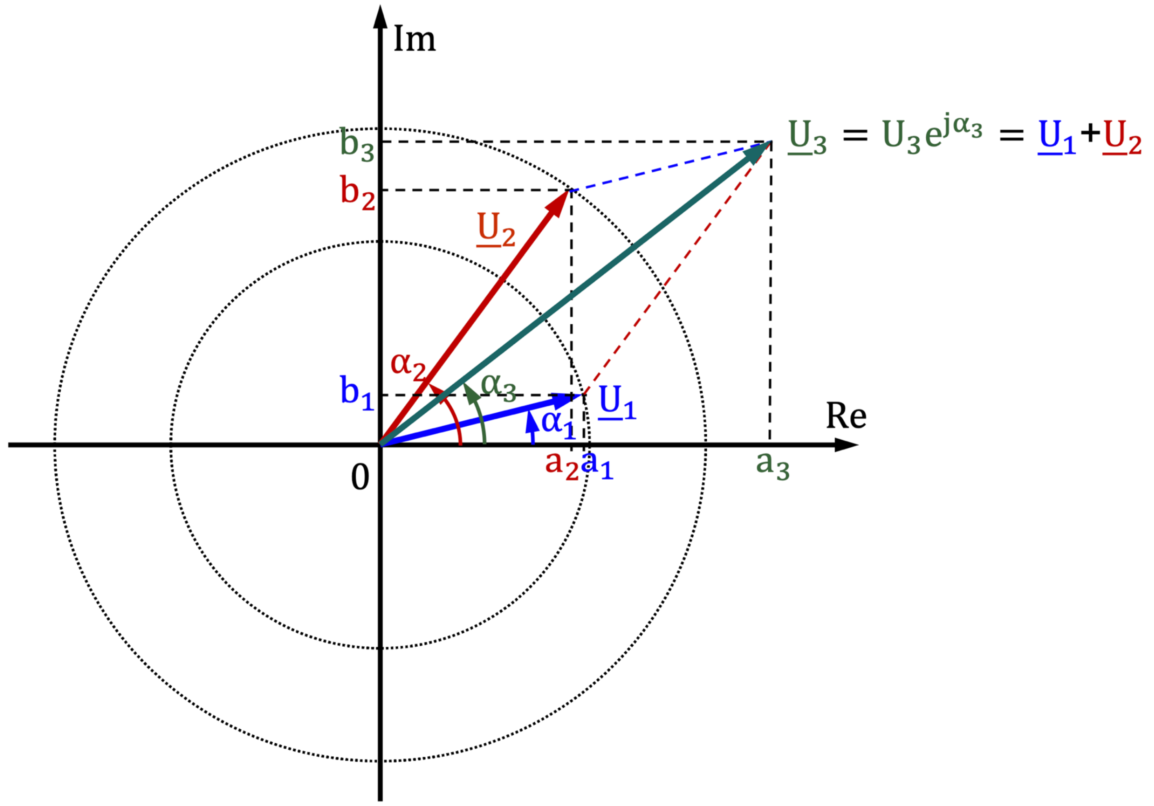
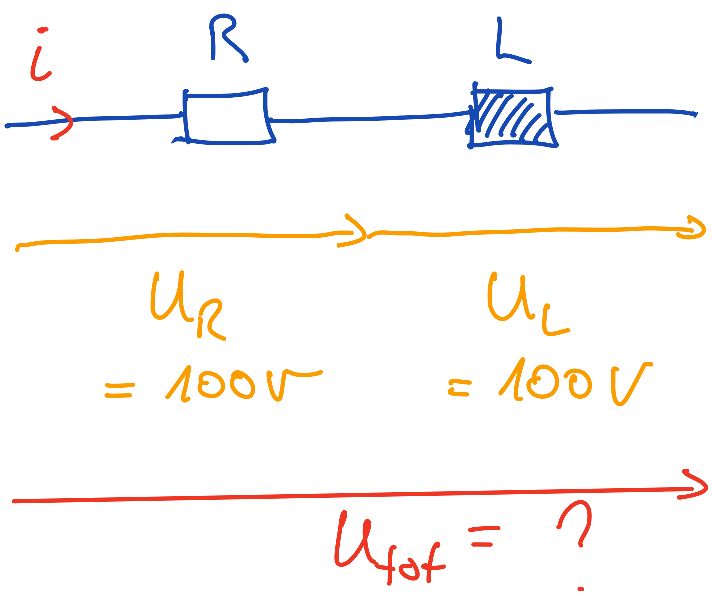


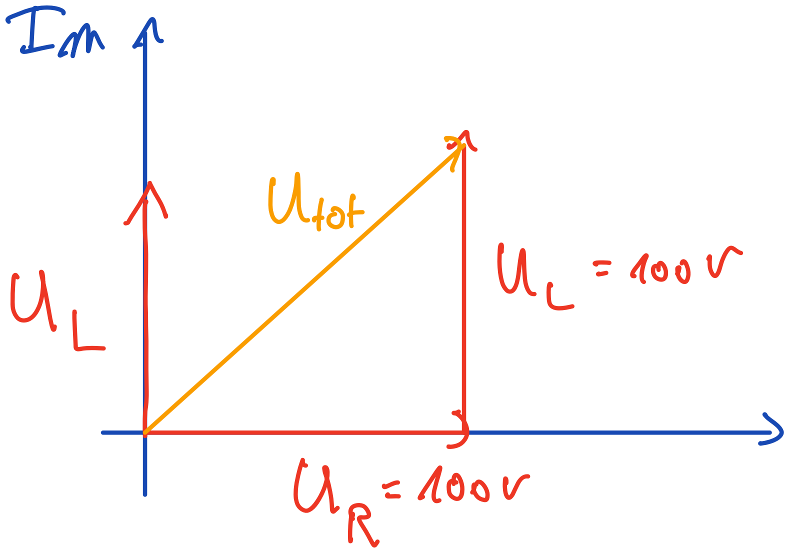
Diagramme des phaseurs



Exemple :



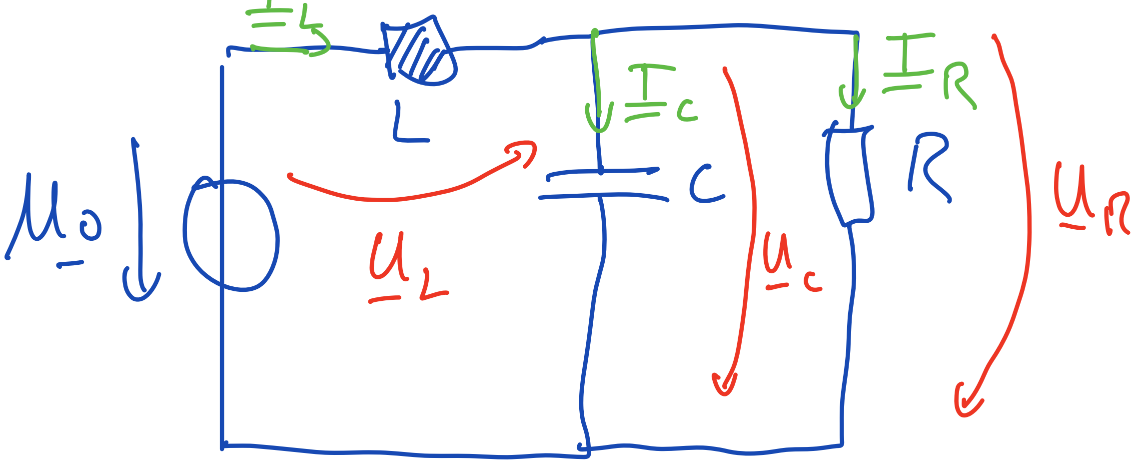
$$u = L \frac{di}{dt}$$



$\underline{U}_L = j\omega L \underline{I}$

$$|\underline{U}_{tot}| = \sqrt{U_R^2 + U_C^2}$$
$$= 141\ V$$

2ième Exemple :



$\underline{I}_c$ = connu $\qquad \underline{U}_0$?

$$\underline{U}_c = \underline{Z}_c \cdot \underline{I}_c = \frac{1}{j\omega C} \cdot \underline{I}_c = -\frac{j}{\omega C} \cdot \underline{I}_c$$

$$\frac{1}{j\omega C} \cdot \frac{j}{j} = \frac{-j}{\omega C}$$

$$\underline{U}_C = \underline{U}_R \qquad \underline{U}_R = R \cdot \underline{I}_R$$

$$\underline{I}_R = \frac{\underline{U}_C}{R} = -j\frac{1}{\omega C R} \cdot \underline{I}_C$$

$$\underline{I}_L = \underline{I}_R + \underline{I}_C$$

$$\underline{I}_L = \underline{I}_C \left(1 - j\frac{1}{\omega C R} \right)$$

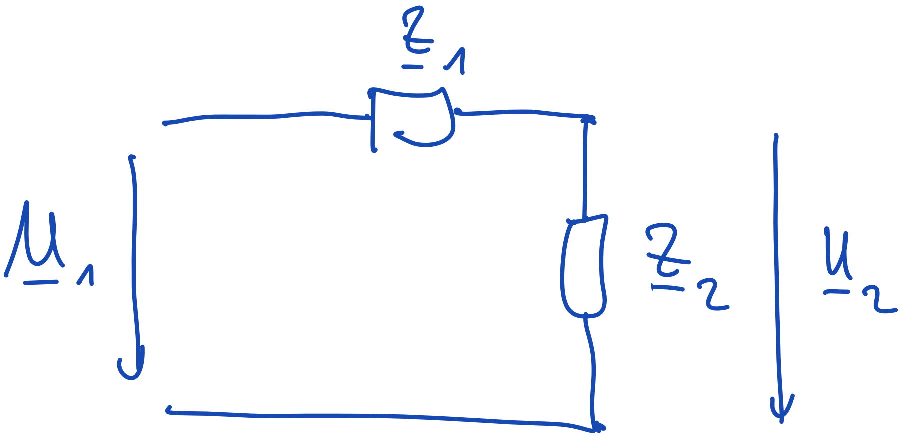
$$\underline{U}_L = \underline{Z}_L \cdot \underline{I}_L = j\omega L \, \underline{I}_L =$$

$$\underline{I}_C \left(\frac{L}{RC} + j\omega L \right)$$

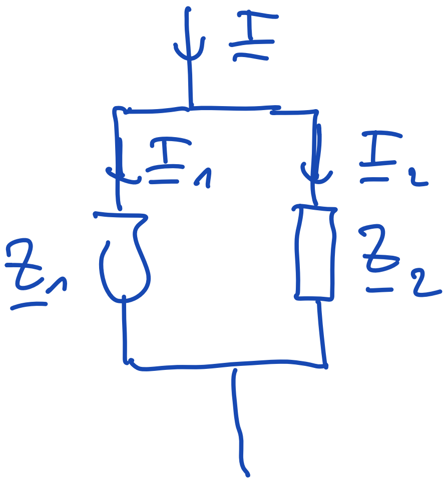
$$\underline{U}_0 = \underline{U}_L + \underline{U}_C = \underline{I}_C \left(\frac{L}{RC} + j\omega L \right) - j\frac{1}{\omega C}\underline{I}_C$$

$$= \underline{I}_C \left[\frac{L}{RC} + j\left(\omega L - \frac{1}{\omega C} \right) \right]$$

7.2.5 Division de tension et courant :



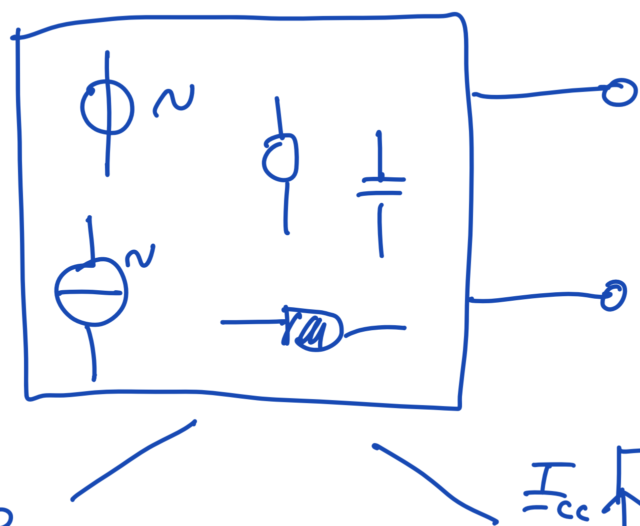
$$\underline{U}_2 = \frac{\underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2} \cdot \underline{U}$$

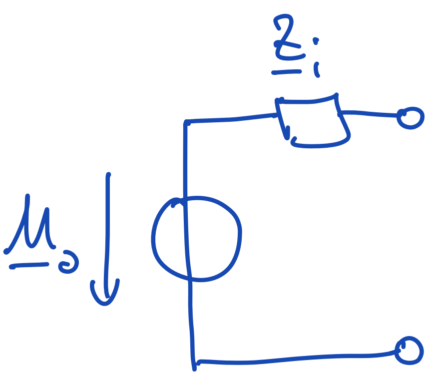


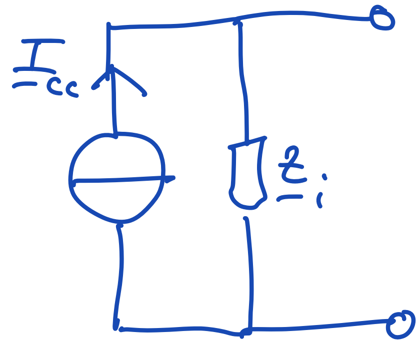
$$\underline{I}_2 = \frac{\underline{Z}_1}{\underline{Z}_1 + \underline{Z}_2} \cdot \underline{I}$$

7.3.1 Théorèmes de Thévenin et Norton

Dipôle :







$\underline{U}_0$ = Tension à vide du circuit

$\underline{I}_{cc}$ = courant de court-circuit

$$\underline{Z}_i = \frac{\underline{U}_0}{\underline{I}_{cc}}$$

Sources de même fréquence

7.4 Principe de Superposition :

Système doit être linéaire

Cas No 1 Toutes les sources ont la même fréquence

On considère chaque source séparément : en annulant les autres :

Source No 1 : $\rightarrow \underline{I}_1$

Source No 2 : $\rightarrow \underline{I}_2$

$\vdots$

$$\underline{I}_{tot} = \sum_{j=1}^{K} \underline{I}_j \qquad K = \text{nb de source}$$

valable pour U et I

Cas No 2 : Les sources n'ont pas la même fréquence !

$f_1 : \longrightarrow \underline{I}_{tot\,1}$

$f_2 : \longrightarrow \underline{I}_{tot\,2}$

$\vdots$

$\underline{I}_{tot\,1} \Rightarrow \underline{i}_{tot\,1} = \sqrt{2}\, \underline{I}_{tot\,1}\, e^{j(\omega_1 t + \beta_1)}$

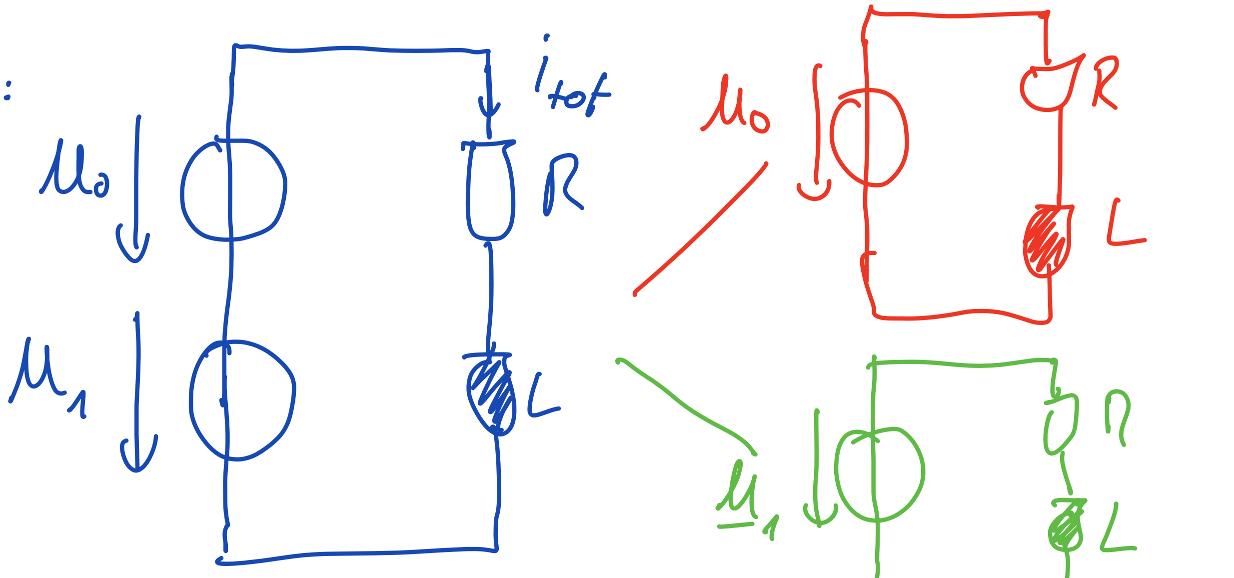
$\underline{I}_{tot\,2} \Rightarrow \underline{i}_{tot\,2} = \sqrt{2}\, \underline{I}_{tot\,2}\, e^{j(\omega_2 t + \beta_2)}$

$\vdots$

$\underline{i}_{tot} = \underline{i}_{tot\,1} + \underline{i}_{tot\,2}$ (monde temporel)

$i_{tot} = \sqrt{2}\, I_{tot\,1} \sin(\omega_1 t + \beta_1) + \sqrt{2}\, I_{tot_2} \sin(\omega_2 t + \beta_2)$

Exemple :

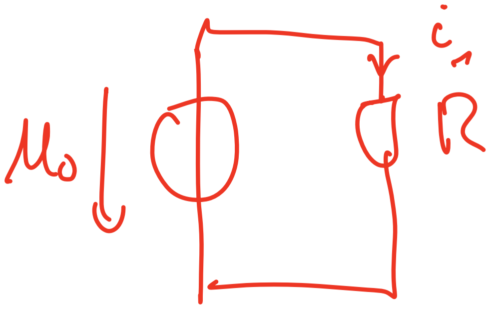


Cas de la source continue U_0 :

$$f = 0 \qquad \omega = 0$$

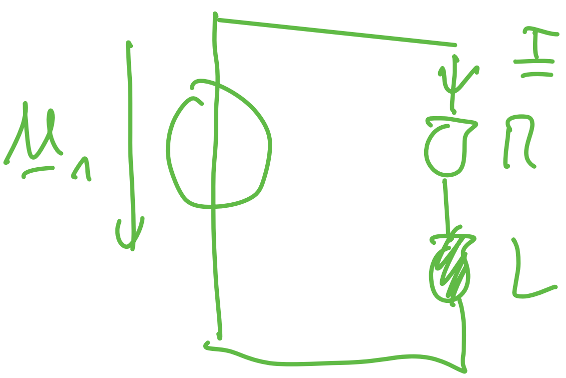
$$\underline{Z}_R = R$$

$$\underline{Z}_L = j\omega L = 0$$



$$i_1 = \frac{U_0}{R}$$

Cas de la source $\underline{U}_1$:



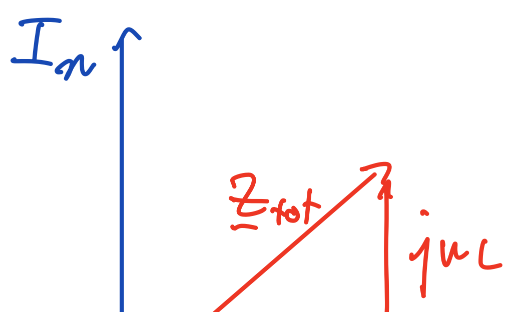
$$\underline{Z}_R = R$$

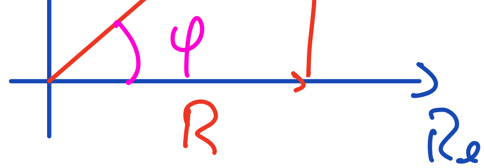
$$\underline{Z}_L = j\omega L$$

$$\underline{Z}_{tot} = R + j\omega L$$

$$\underline{U}_1 = \underline{Z}_{tot} \cdot \underline{I}_2$$

$$|\underline{Z}_{tot}| = \sqrt{R^2 + \omega^2 L^2}$$





$$\varphi = \operatorname{Arctg} \frac{\omega L}{R}$$

$$\underline{U}_1 = U_1 e^{j(0)} \qquad (\text{On pose } \alpha = 0)$$

$$= U_1$$

$$\underline{I}_2 = \frac{\underline{U}_1}{\underline{Z}_{tot}} = \frac{U_1}{\sqrt{R^2+\omega^2L^2}} \frac{e^{j0}}{e^{j\varphi}} = I_2 e^{j-\varphi}$$

$$\Rightarrow$$

$$\underline{i}_2 = \sqrt{2}\, I_2\, e^{j(\omega t - \varphi)}$$

Monde réel : $i_2 = \sqrt{2}\, I_2 \sin(\omega t - \varphi)$

Résultat final : $i_{tot} = \frac{U_0}{R} + \sqrt{2}\, I_2 \sin(\omega t - \varphi)$

8. Puissances en alternatif sinus monophasé :

8.1 Puissance instantanée :

$$P(t) = p = u \cdot i$$

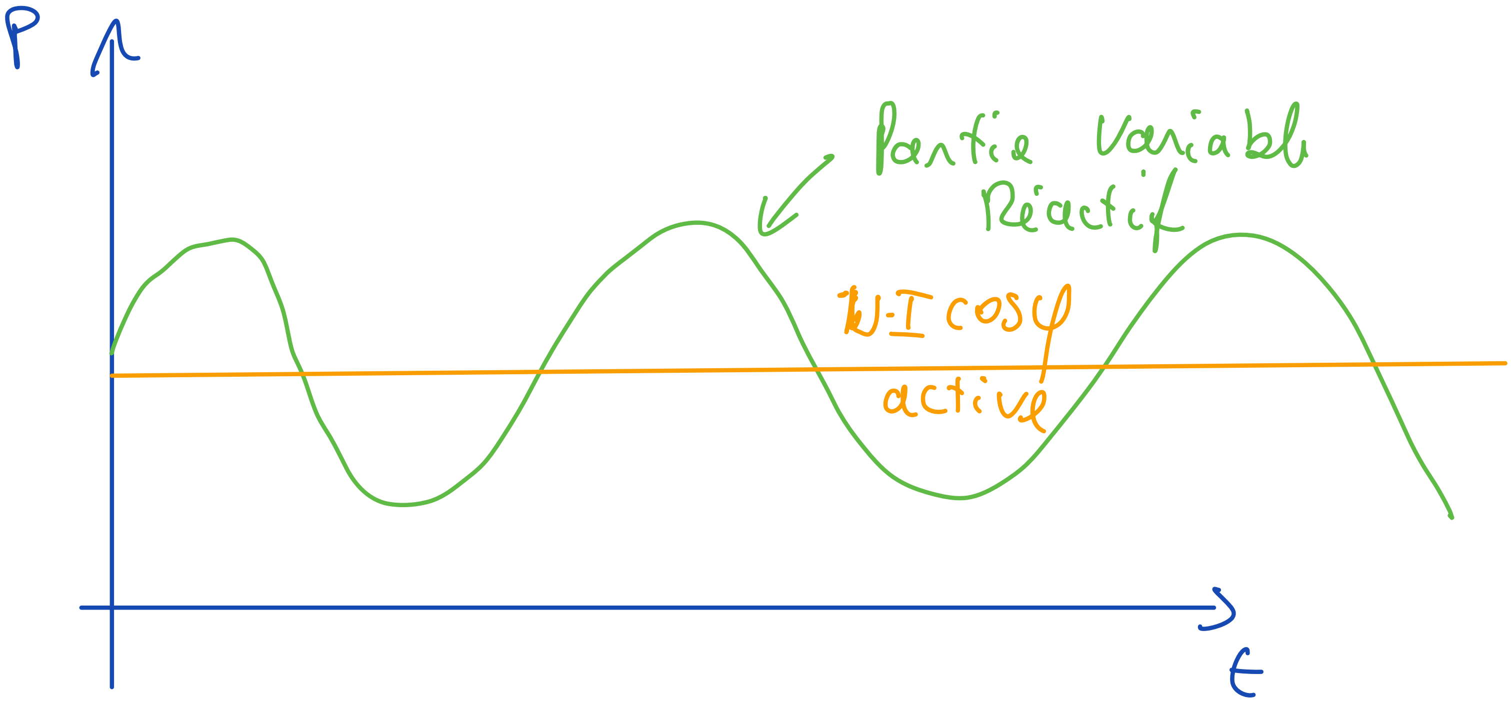
$$u = \hat{U} \cos(\omega t + \alpha)$$
$$i = \hat{I} \cos(\omega t + \beta)$$

$$p = \hat{U}\hat{I} \cos(\omega t + \alpha) \cos(\omega t + \beta)$$

$$\cos x \cdot \cos y = \frac{1}{2}\left(\cos(x-y) + \cos(x+y)\right)$$

$$p = \frac{\hat{U}\hat{I}}{2}\left[\cos\underbrace{(\alpha-\beta)}_{\varphi} + \cos(2\omega t + \alpha + \beta)\right]$$

$$* \quad p = U \cdot I\left[\cos\varphi + \cos(2\omega t + \alpha + \beta)\right]$$



Impédance : $\underline{Z} = R + jX$

$R \downarrow$ Résistance

$X \downarrow$ réactance

$L \rightarrow \underline{Z}_L = j\underbrace{\omega L}_{X_L}$

On pose $\beta = \alpha - \varphi$

Identité : $\cos(2\omega t + 2\alpha - \varphi)$

$$= \cos\varphi \cos(2\omega t + 2\alpha) + \sin\varphi \sin(2\omega t + 2\alpha)$$

$$* \; p(t) = \underbrace{UI\cos\varphi\left[1 + \cos(2\omega t + 2\alpha)\right]}_{a}$$

$$+ \underbrace{UI\sin\varphi\sin(2\omega t + 2\alpha)}_{b} \quad \text{Eq. } 8.3$$

a : oscille autour de $UI\cos\varphi$
toujours positif

b : oscille autour de 0, amplitude
moyenne est nulle

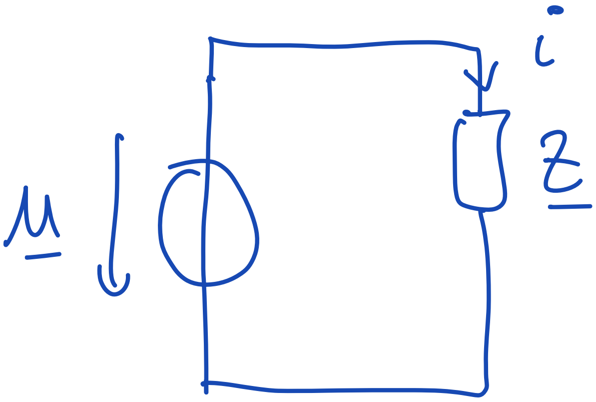
8.2 Puissance Active :

$$P = \overline{p}(t) = \frac{1}{T}\int_0^T p(t)\,dt$$

$$= UI\cos\varphi \quad [W]$$

P = valeur moyenne de $p(t)$

= ce que l'on transforme et paye !



$$\varphi = \operatorname{Arctg} \frac{X}{R}$$

a) $\underline{Z} = R \rightarrow \varphi = 0$

$$P_R = UI\cos\varphi = UI$$

$$= \frac{\hat{U}\hat{I}}{2} = R \cdot I^2$$

b) Si $\underline{Z}_L = L$

$$\underline{Z}_L = j\omega L \qquad \varphi_L = \frac{\pi}{2}$$

$$P_L = UI\cos\varphi = 0$$

c) Si $\underline{Z}_c \Rightarrow C$

$$\underline{Z}_c = -\frac{j}{\omega C} \qquad \varphi_c = -\frac{\pi}{2}$$

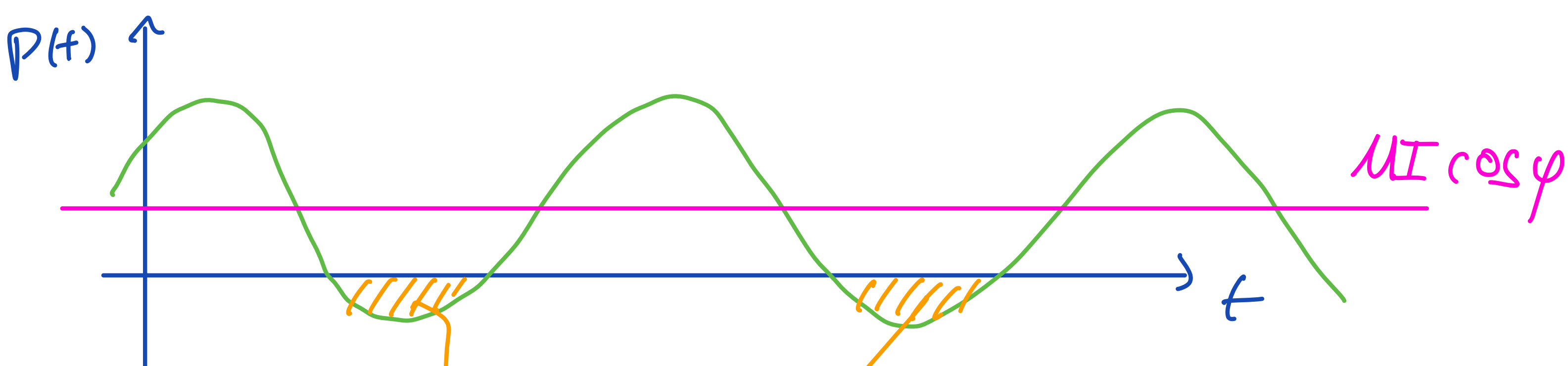
$$P_c = UI\cos\varphi = 0$$

8.3 Puissance Réactive :

Par définition, Amplitude de la composante Alternative de $p(t)$

Puissance fictive → caractérise l'échange de puissance non convertible

$$Q = UI\sin\varphi \quad [\text{Var}]$$



p rendu
à la source

Réactive positif : pour une inductance
" négatif : pour une capacité

Rappel : $P = UI \cos\varphi$

$Q = UI \sin\varphi$

8.4 Puissance Apparente :

$S = U \cdot I$ [VA]

~ Valeur de l'objet
~ prix de l'objet.

$P = S \cos\varphi$

$Q = S \sin\varphi$

$$S = UI$$

$$S = \sqrt{P^2 + Q^2}$$

8.5.3 Facteur de Puissance :

$$P = UI \underbrace{\cos\varphi}$$

facteur de puissance

$$\cos\varphi = \frac{P}{UI}$$

8.5 Puissance apparente complexe

$$\underline{S} = P + jQ = Se^{j\varphi}$$

$$P = S\cos\varphi$$

$$Q = S\sin\varphi$$

$$S = UI = \sqrt{P^2 + Q^2}$$

$P = UI \cos\varphi$ $\quad R \rightarrow P_R = R \cdot I^2$

$Q = UI \sin\varphi$ $\quad L, C \rightarrow Q_C = \underset{\text{réactance}}{X} \cdot I^2$

Si L : $X = \omega L$

C : $X = -\dfrac{1}{\omega C}$

$$\underline{S} = \underbrace{RI^2}_{P} + \underbrace{jXI^2}_{Q}$$

8.5 - 4 - 6 :

R : $\quad \underline{Z} = R \quad \varphi = 0$

$$P_R = UI = RI^2 = \frac{U^2}{R}$$

$$Q_R = 0$$

$$S_R = UI = P_R$$

$$\cos\varphi = 1$$

L :

$$\underline{Z} = j\omega L \qquad \varphi = \frac{\pi}{2}$$

$$P_L = 0$$

$$Q_L = UI = XI^2 = \omega L I^2$$

$$S_L = UI$$

$$\cos\varphi = 0$$

C :

$$\underline{Z}_c = \frac{1}{j\omega c} = -\frac{j}{\omega c} \qquad \varphi = -\frac{\pi}{2}$$

$$P_c = 0$$

$$Q_c = -UI = -\frac{1}{\omega c} \cdot I^2$$

$$S_c = UI$$

$$\cos\varphi = 0$$

8.6 Résolution par les puissances :

Propriété :

$$P_{tot} = \sum_{k=1}^{n} P_k$$

n = nb de composants

$$Q_{tot} = \sum_{k=1}^{n} Q_k$$

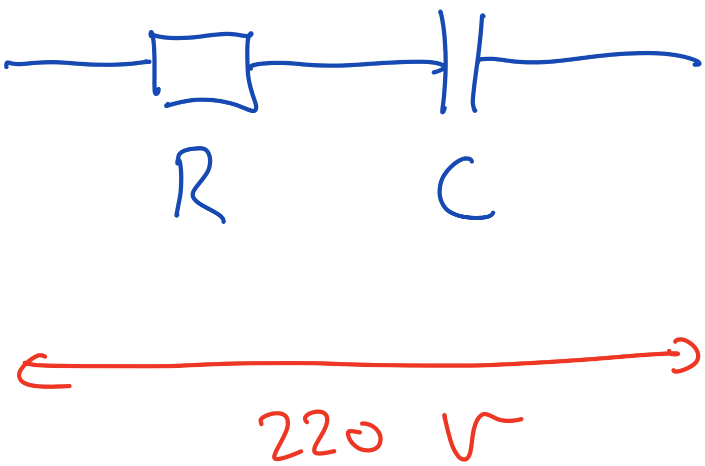
n = nb de composants

Mais : ⚠ $S_{tot} \neq \sum_{k=1}^{n} S_k$

Mais : $\underline{S}_{tot} = \sum_{k=1}^{n} \underline{S}_k$

(vectorielle)

Exemple :



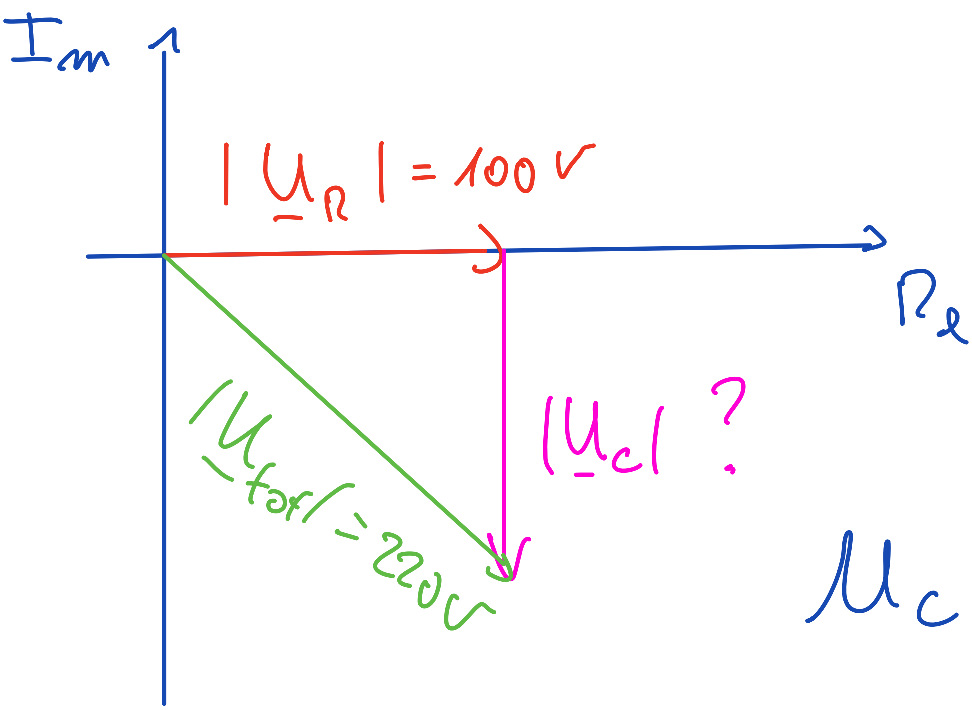
$f = 50\ Hz$

$U_R = 100\ V$

$P_{tot} = 500\ W$

U_C ?

$\underline{S}$?



$$U_C = \sqrt{U_{tot}^2 - U_R^2}$$

$$= 195{,}95\ V$$

$$\underline{S} = P + jQ$$

$$P = P_R = P_{tot} = 500\ W = R I^2$$

$$= \frac{U_R^2}{R}$$

$$\rightarrow \quad R = 20\ \Omega$$

$$\rightarrow \quad I = 5A$$